Microprocessor-based protective relays

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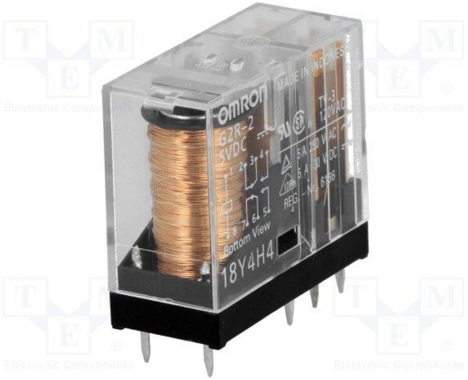
introduction

- The increased growth of power system both in size and complexity has brought about the need for fast and reliable relays to protect major equipment and to maintain system stability.

- The conventional protective relays :

electromagnetic



static

- The electromagnetic relays have several drawbacks:

1. high burden on instrument transformers
2. high operating time
3. contact problems
4. Etc.

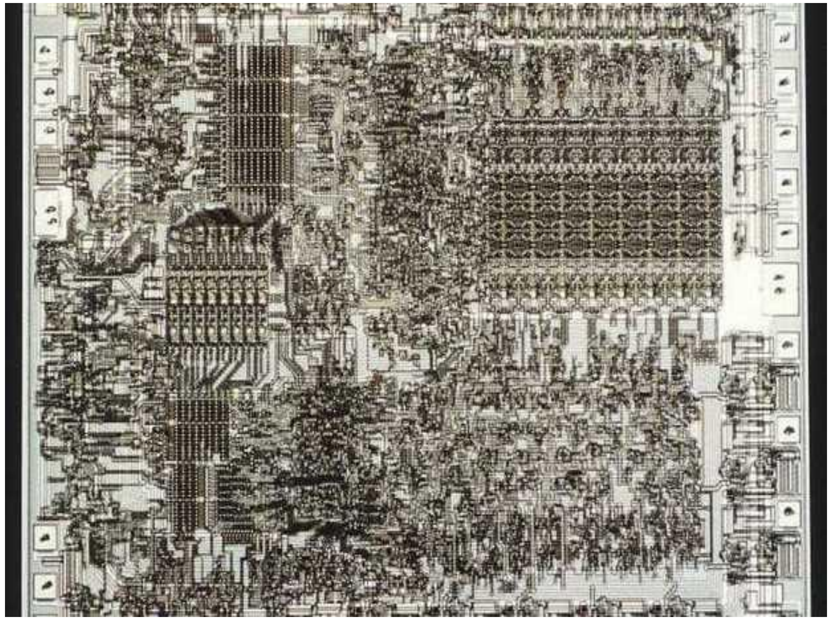
- static relay features:

1. Compactness
2. lower burden
3. less maintenance
4. high speed

- several drawbacks of static relay:

1. Inflexibility
2. inadaptability to changing system conditions and complexity

- The advent of microprocessors in the 1970s initiated a revolution in the design and development of digital protection schemes.



- With the development of economical, powerful and sophisticated microprocessors, there is a growing interest in developing microprocessor-based protection relays which are more flexible because of being programmable and are superior to conventional electromagnetic and static relays.

- The main features which have encouraged the design and development of microprocessor-based protective relays are:

1. Economy
2. Compactness
3. Reliability
4. Flexibility
5. Improved performance over conventional relays

- A number of desired relaying characteristics, such as overcurrent, directional, impedance, reactance mho, quadrilateral, elliptical, etc. can be obtained using the same interface.
- Using multiplexer, a microprocessor can get the desired signals to obtain a particular relaying characteristic.

- The relaying signal i.e. fault current and voltage contain harmonics and d.c. offset components which require filtering before feeling them to the relay.

- The microprocessor-based relays which assume sinusoidal waves for the fault voltage and current use analog band-pass filters which pass 40-70 Hz signals.

- But at low frequency, the analog filters are slow. Research is in progress to develop faster relays using digital filter techniques.

Overcurrent Relays

- An overcurrent relay is the simplest form of protective relay which operates when the current in any circuit exceeds a certain predetermined value, i.e. the pick-up value.
- Using a multiplexer, the microprocessor can sense the fault currents of a number of circuits. If the fault current in any circuit exceeds the pick-up value, the microprocessor accepts signals in voltage form, the current signal derived from the current transformer is converted into a proportional voltage signal using a current to voltage converter. The a.c. voltage proportional to the load current is converted into d.c. using a precision rectifier. Thus, the microprocessor accept d.c. voltage proportional to the load current.
- The output of the rectifier is fed to the multiplexer. The microcomputer sends a command to switch on the desired channel of the multiplexer to obtain the rectified voltage proportional to the current in a particular circuit. The Microcomputer reads the end of conversion signal to examine whether the conversion is over or not. As soon as the conversion is over, the microcomputer reads the current signal in digital form and then compares it with the pick-up value.

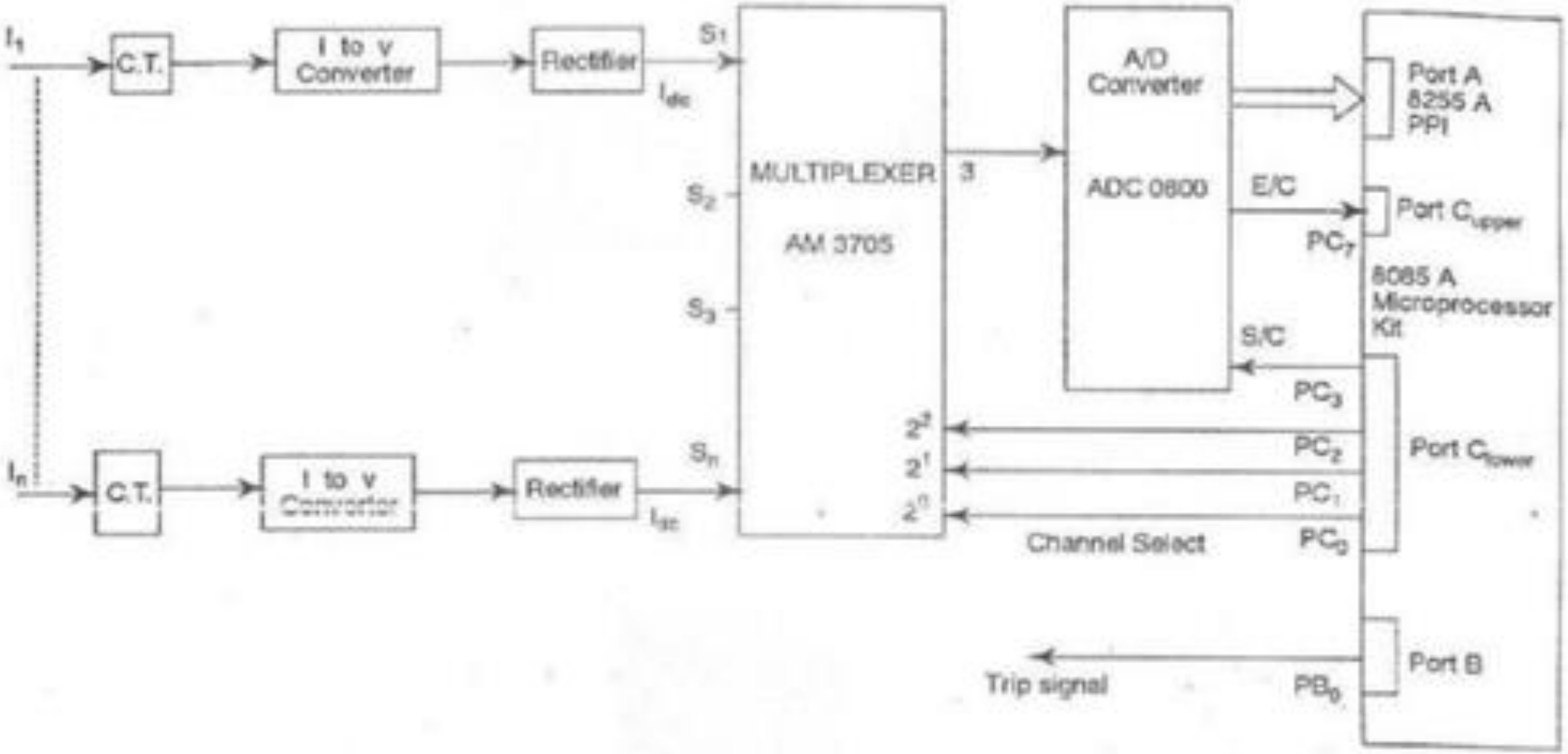


FIGURE 8.1(a) Block schematic diagram of overcurrent relay

- In the case of definite time overcurrent relay, the microcomputer sends the tripping signal to the circuit breaker after a predetermined time delay if the fault current exceeds the pick-up value. In case of instantaneous-time characteristics, the operating times for different values of currents are noted for a particular characteristic. These values are stored in the memory in tabular form. The microcomputer first determines the magnitude of the fault current and then selects the corresponding time of operation from the look-up table. A delay subroutine is started and the tripe signal is sent after the desired delay. Using the same program any characteristic such as IDMT, very inverse or extremely inverse can be realized by simply changing the data of the look-up table according to the desired characteristic to be realized. The microcomputer continuously measures the current and moves in a loop and if the measured current exceeds the pick-up value, it compares the measured value of the current with the digital values of current given in the look-up table in order to select the corresponding count for a time delay. Then it goes in delay subroutine and sends a trip signal to the circuit breaker after the predetermined time delay.

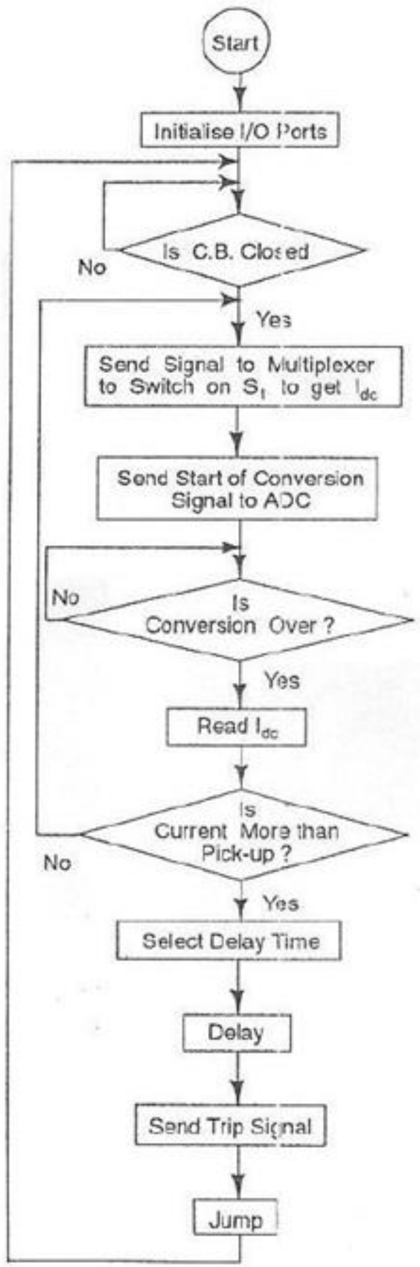


FIGURE 8.1(b) Program flowchart for overcurrent relay

- In order to avoid false tripping of an overcurrent relay due to transients the program can be modified slightly. When the fault current exceeds the pick-up values, the fault current is measured once again by the microprocessor to confirm whether it is a fault current or transient. In case of any transient of short duration, the measured current above pick-up value will not appear in the second measurement. But if there is an actual fault, it will again appear in the second measurement also, and then the microprocessor will issue a tripping signal to disconnect the faulty part of the system.

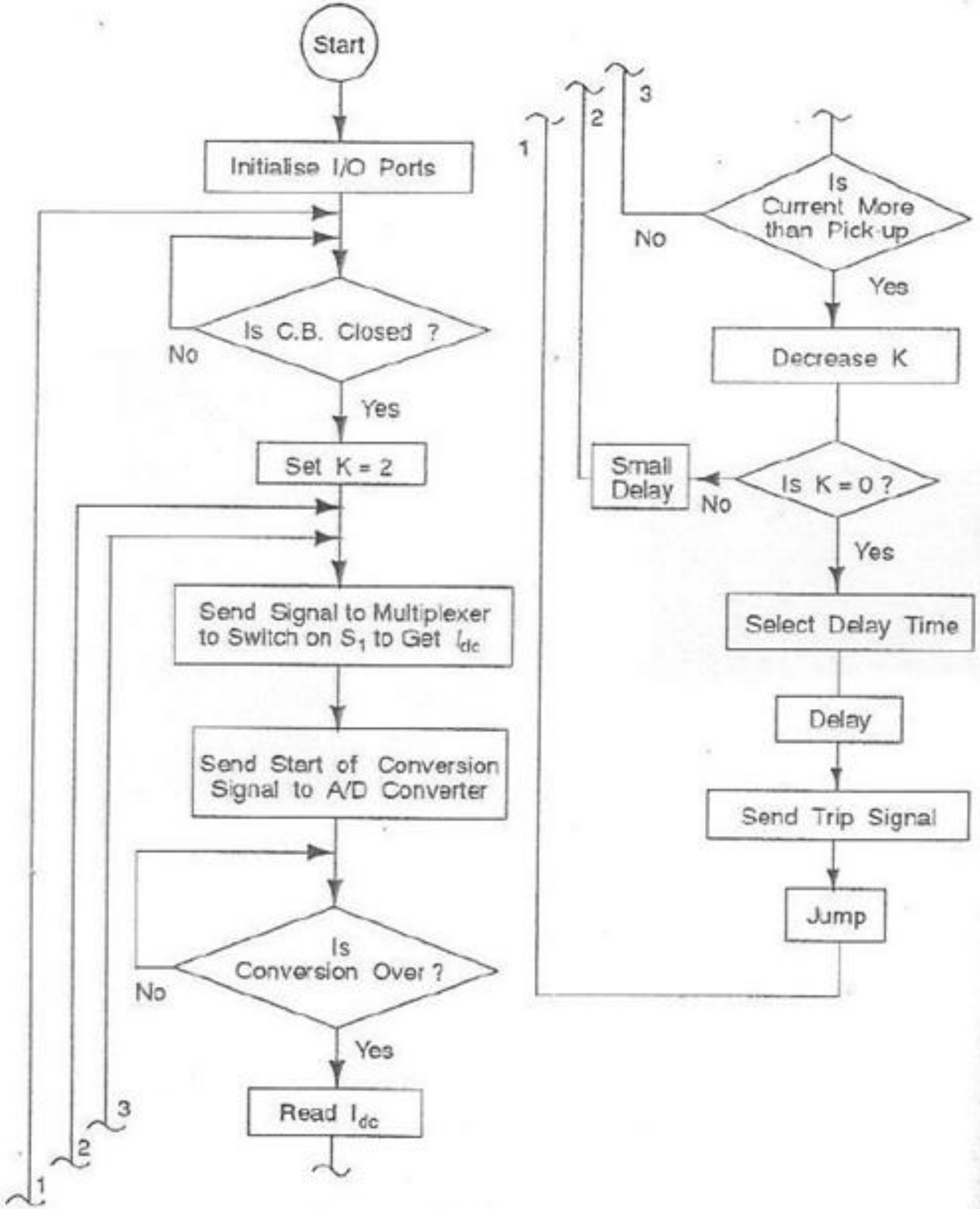


FIGURE 8.2 Program flowchart for overcurrent relay

impedance relay

- The characteristic of an impedance relay is realized by comparing voltage and current at the relay location. The ratio of voltage (v) to current (I) gives the impedance of the line section between the relay location and fault point. The rectified voltage (V dc) and rectified current (I dc) are proportional to V and I, respectively. Therefore, for comparison V dc and I dc are used. The following condition should be satisfied for the operation of the relay.

$$K_1 V_{dc} < k_2 I_{dc} \text{ OR } \frac{V_{dc}}{I_{dc}} < \frac{K_2}{K_1}$$

$$\frac{V}{I} < K$$

$$Z < K$$

- As the impedance relay is non-directional, a directional unit is also incorporated to given directional, a directional unit is also incorporated to give a directional feature so that the relay can operate for the fault in forward direction only.
- The levels of voltage and current signals are stepped down to the electronic level by using potential and current transformers. The current signal derived from the current transformer is converted into proportional voltage signal using a current to voltage converter. The voltage and current signals are then rectified using precision rectifiers to convert them into dc. The rectified voltage and current signal (V dc and I dc respectively) are fed to two different channels of the multiplexer which are switched on sequentially by proper commands from the microcomputer. The output of the multiplexer is fed to the A/D converter through a sample and hold circuit. The multiplexer (AM3705), sample and hold (LF398) and 8-bit A/D converter ADC0800 form the data acquisition system (DAS). The data acquisition system (DAS) is interfaced to the microprocessor using 8255A programmable peripheral interface. A clock of 300 kHz for ADC0800 is obtained by dividing the 3 MHz clock of the microprocessor by ten using the IC package 7490. Suitable clock for ADC can also be obtained by using programmable timer/counter, intel 8253, the controls for analog multiplexer, sample and hold, and ADC are all generated by the microcomputer under program control.

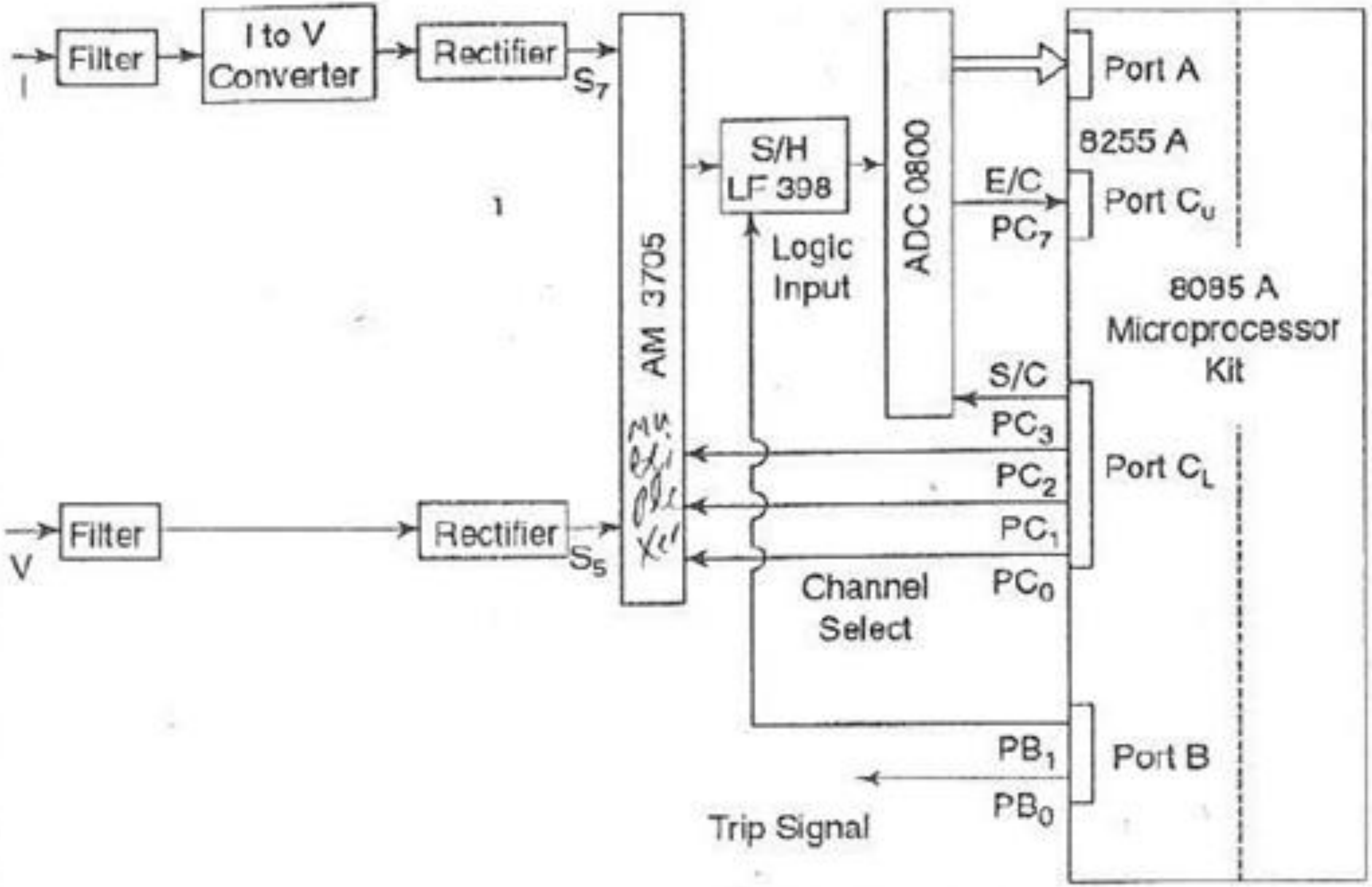


FIGURE 8.3 Block schematic diagram of interface for impedance relay

- The microcomputer reads V_{dc} and I_{dc} , calculates the impedance Z seen by the relay and then compares Z with Z_1, i.e. the predetermined value of impedance for the first zone of protection. If Z is less than Z_1, the microcomputer sends a tripping signal to the trip coil of the circuit breaker instantaneously. If Z is greater than Z_1, the comparison is made with Z_2, i.e. the value of impedance for the second zone of protection. If Z is less than Z_2, the microcomputer takes up a delay subroutine and sends the trapping signal to the trip coil after a predetermined delay. If Z is greater than Z_2 but less than Z_3, a greater delay is provided before the tripping signal is sent. If Z is more than Z_3, the microcomputer again reads the voltage and current signals and proceeds according to its program.

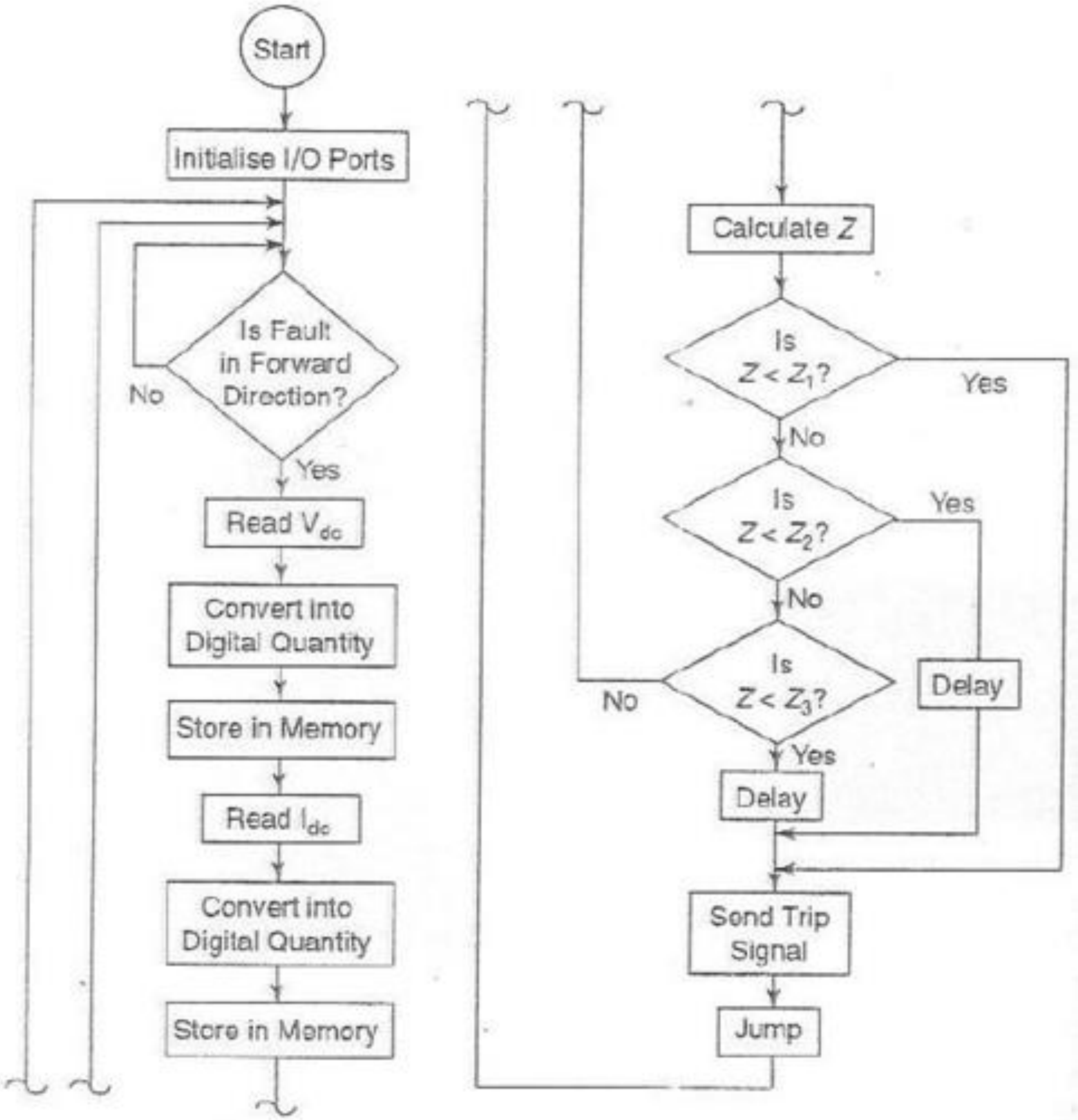


FIGURE 8.4 Program flowchart for impedance relay

directional relay

- A directional relay senses, the direction of power flow. The polarity of the instantaneous value of the current at the moment of the voltage peak is examined to judge the direction of power flow. The program developed for this relay is able to judge whether the fault point is in the forward or reverse direction with respect to relay location as I Fig. 8.5 (a). The instantaneous value of the current at the moment of voltage peak is $I_m \cos\Phi$, as shown in Fig. 8.5 (b). For a fault point lying in the forward direction, $I_m \cos\Phi$, is positive for Q lying within $\pm 90^\circ$.For a fault lying in the reverse direction, $I_m \cos\Phi$ becomes negative

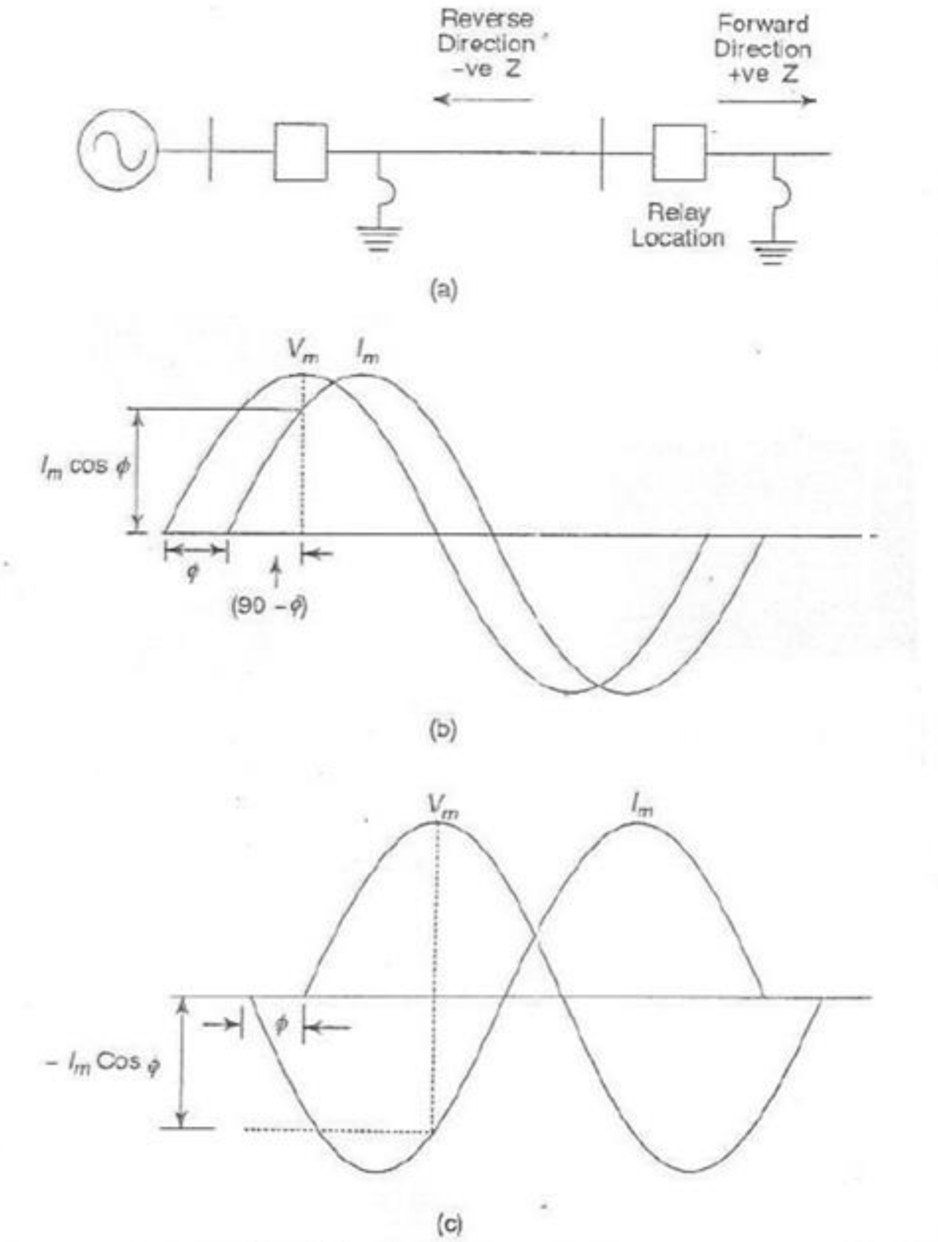


FIGURE 8.5 (a) Location of directional relay (b) Instantaneous value of current at peak voltage (c) $I_m \cos \phi$ for fault in reverse direction

- A phase-shifter and a zero-crossing detector are used to obtain a Pulse at the moment of voltage peak. The voltage signal is fed to the phase-shifter to get a phase-shifter of 90°. then the output of the phase-shifter is fed to a zero-crossing detector to obtain the required pulse. The microcomputer reads the output of the aero-crossing to examine whether the voltage has crossing its peak value. After receiving the pulse the microcomputer sends a command to the multiplexer to switching on the channel S_2 to obtain the microcomputer reads this value of the current through the A/D converter and examines whether it is positive.
- This type of relay can be used to sense the reversal of power flow where it is required. When the power flow is reversed, $I_m \cos\Phi$ becomes negative and the microcomputer sends a tripping signal to the circuit breaker. The program flowchart is shown in Fig. 8.7. This relay can also be used in conjunction with overcurrent relays and impedance relays to provide directional features. When it is used as a directional relay, it other energises other relay only when the fault lies in the forward direction. It can also be used as a direction relay in conjunction with overcurrent relays for the protection of parallel lines.

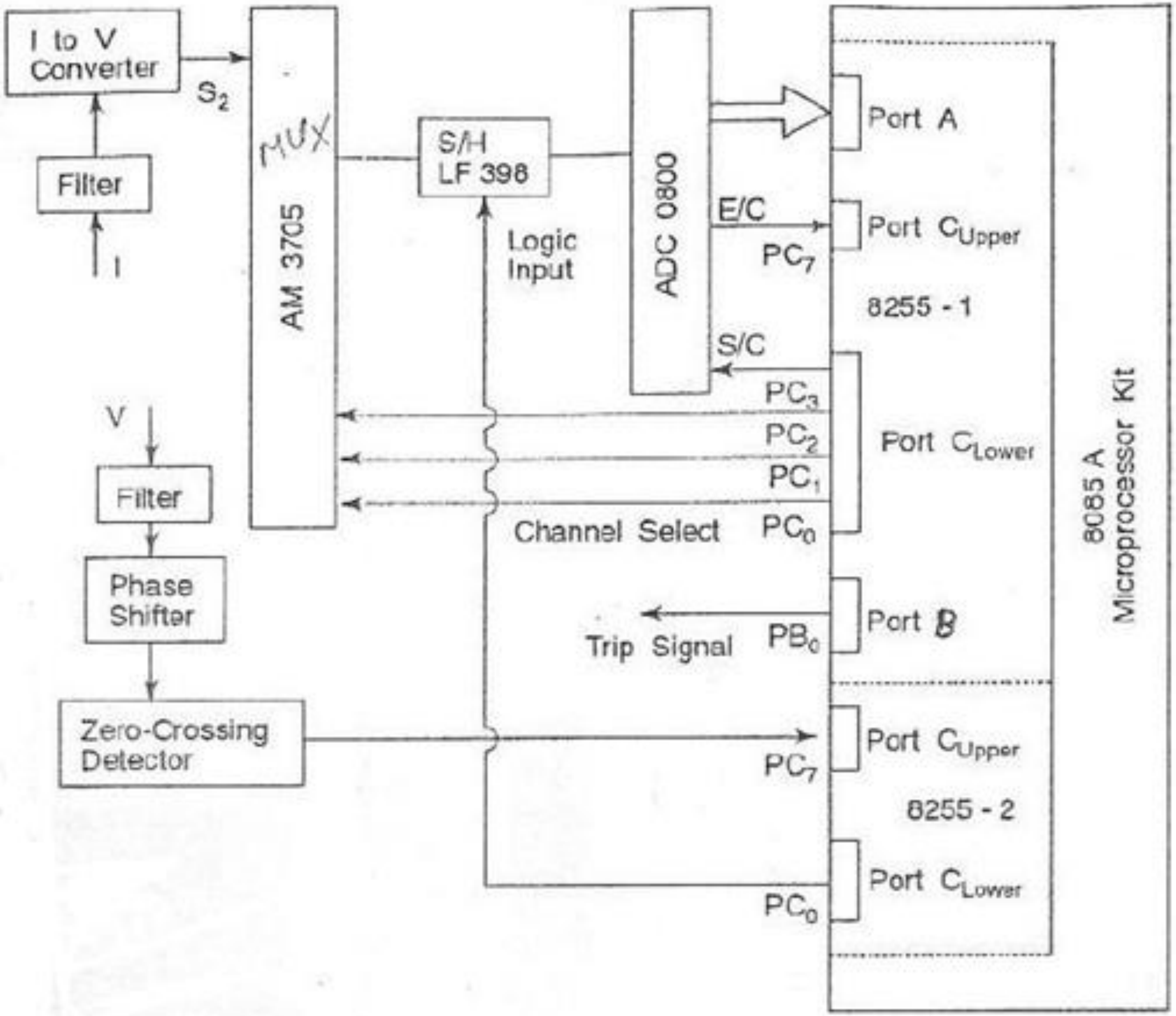


FIGURE 8.6 Block schematic diagram of interface for directional relay

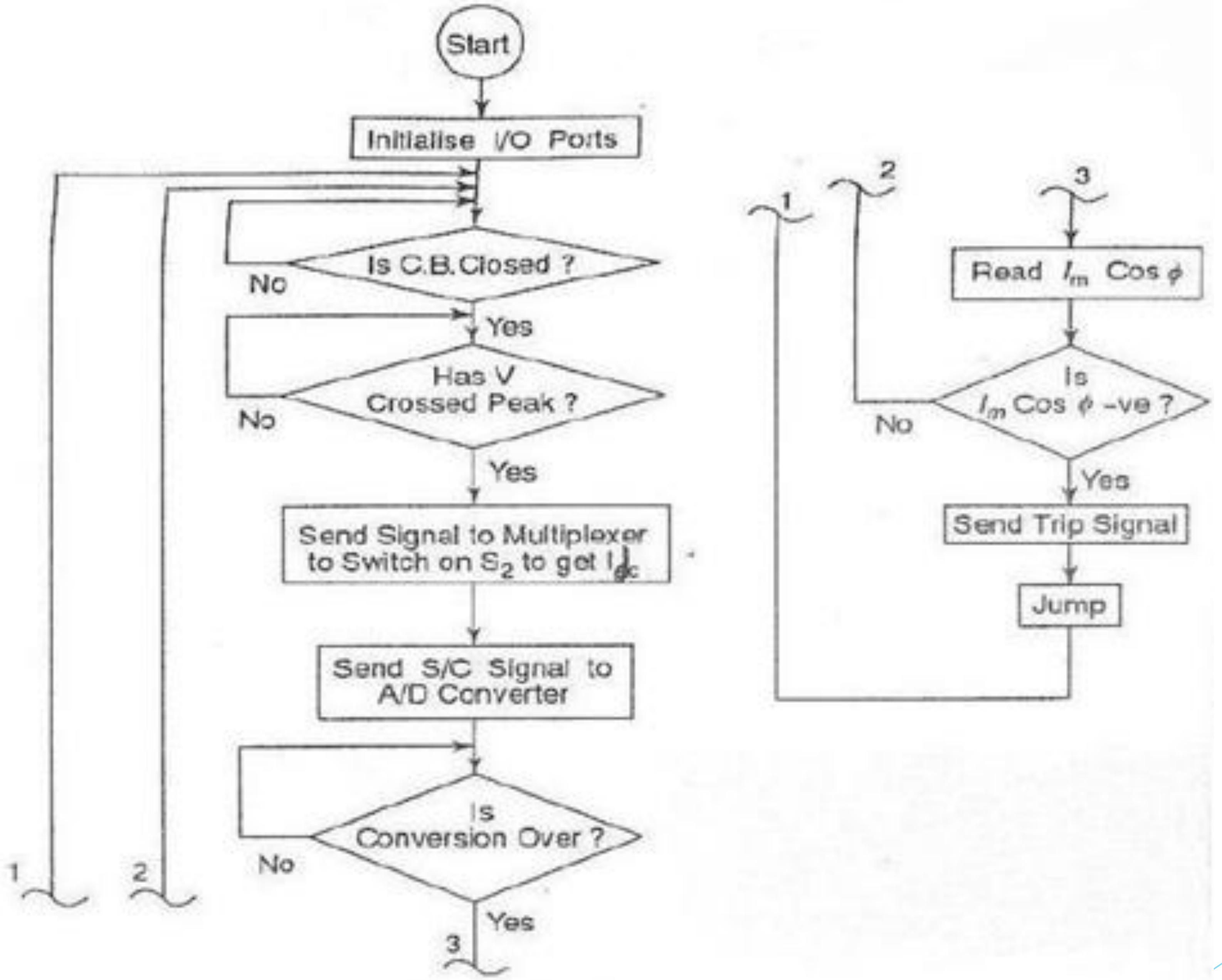


FIGURE 8.7 Program flowchart for reverse power relay

reactance relay

- The characteristic of a reactance relay is realized by comparing the instantaneous value of the voltage at the moment of current zero against the rectified current. The instantaneous value of voltage at the moment of current zero is $V_m \sin\Phi$, as shown in Fig. 8.8. For the operation of the relay, the condition to be satisfied is as follows.

$$V_m \sin\Phi < K_1 I_{dc} \quad \text{or} \quad \frac{V_m \sin\Phi}{I_{dc}} < K_1$$

$$\frac{V \sin\Phi}{I} < K$$ (as V_m and I_{dc} are proportional to rms values V and I, respectively)

$$Z \sin\Phi < K$$

$$X < K$$

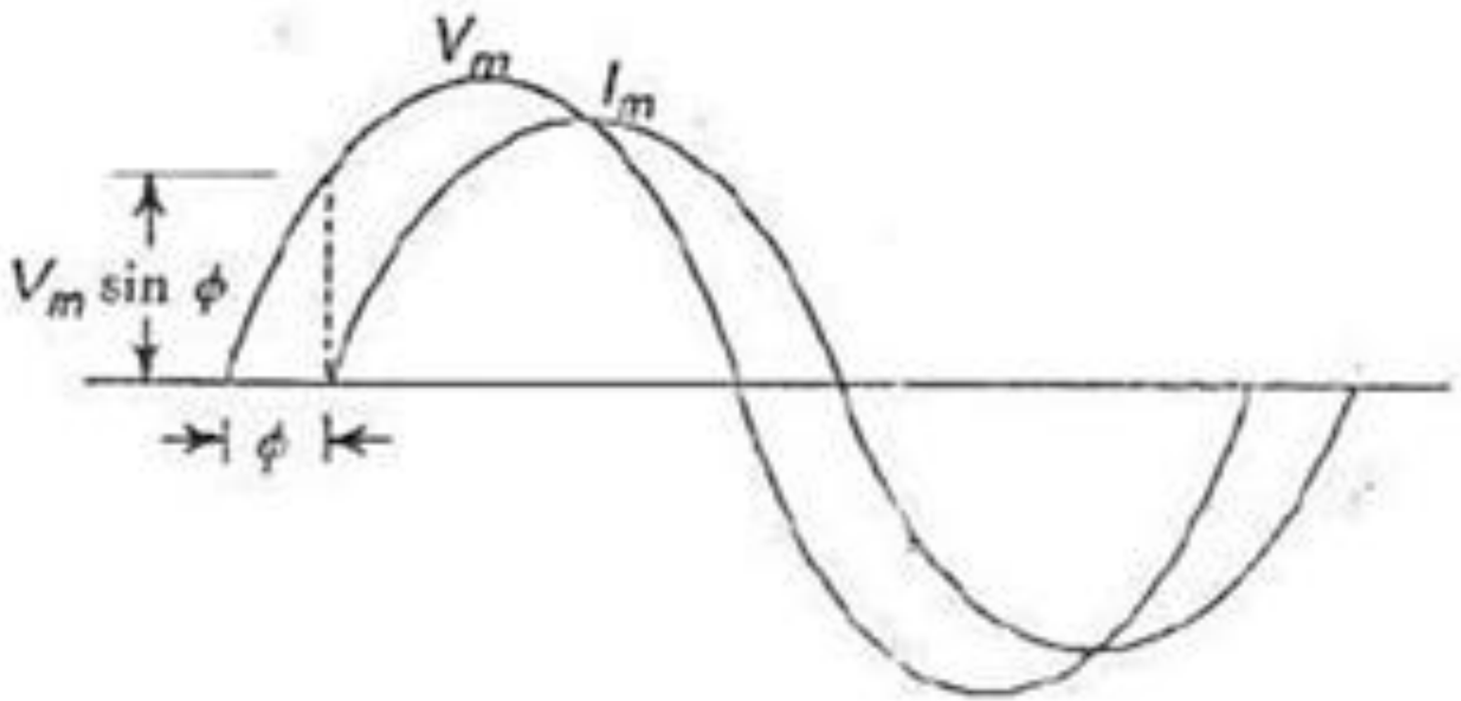


FIGURE 8.8 Instantaneous value of voltage at current zero

- The microcomputer reads the output of the zero crossing detector to examine whether the current has crossed its zero point. As soon as the current crosses is zero point, the microcomputer sends a command to the multiplexer to switch on channel S_4, and gets the instantaneous value of the voltage, i.e. $V_m \sin\Phi$ through the A/D converter. Then the microcomputer sends a command to the multiplexer to switching on the channel S_7 to get the rectified current. Thereafter, the microcomputer calculates X, the reactance as seen by the relay and compares it with X_1, the predetermined value of the reactance for the first zone of protection. The microcomputer sends a tripping signal instantaneously, if the measured value of X is less than X_1. If X is greater than X_1, but less than X_2, the tripping signal is sent after a predetermined delay. If X is more than X_2 but lies within the protection zone of the direction unit which also acts as a third unit as shown in Fig. 8.10, the tripping signal is sent after a greater predetermined delay.

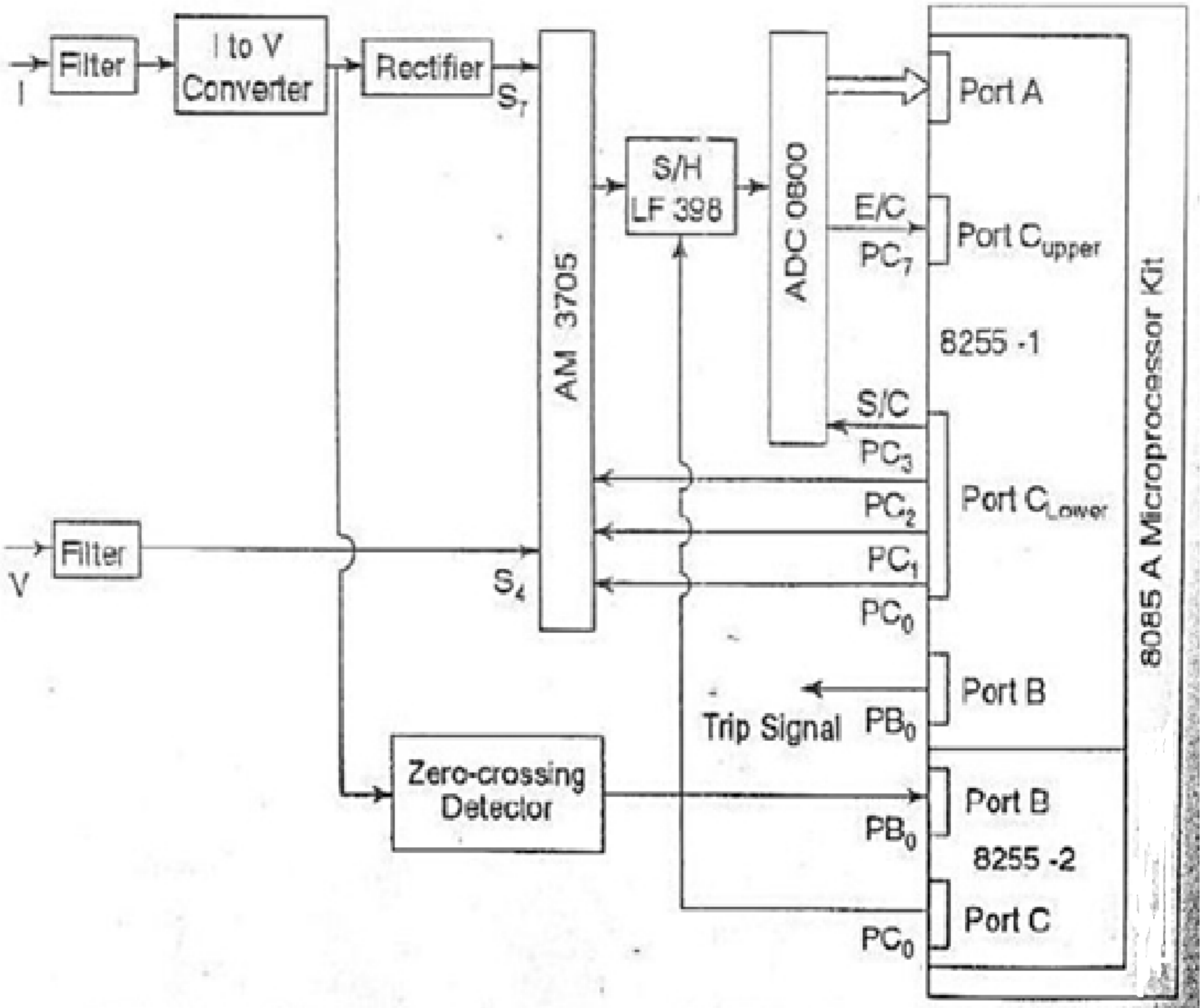


FIGURE 8.9 Block schematic diagram of interface for reactance relay

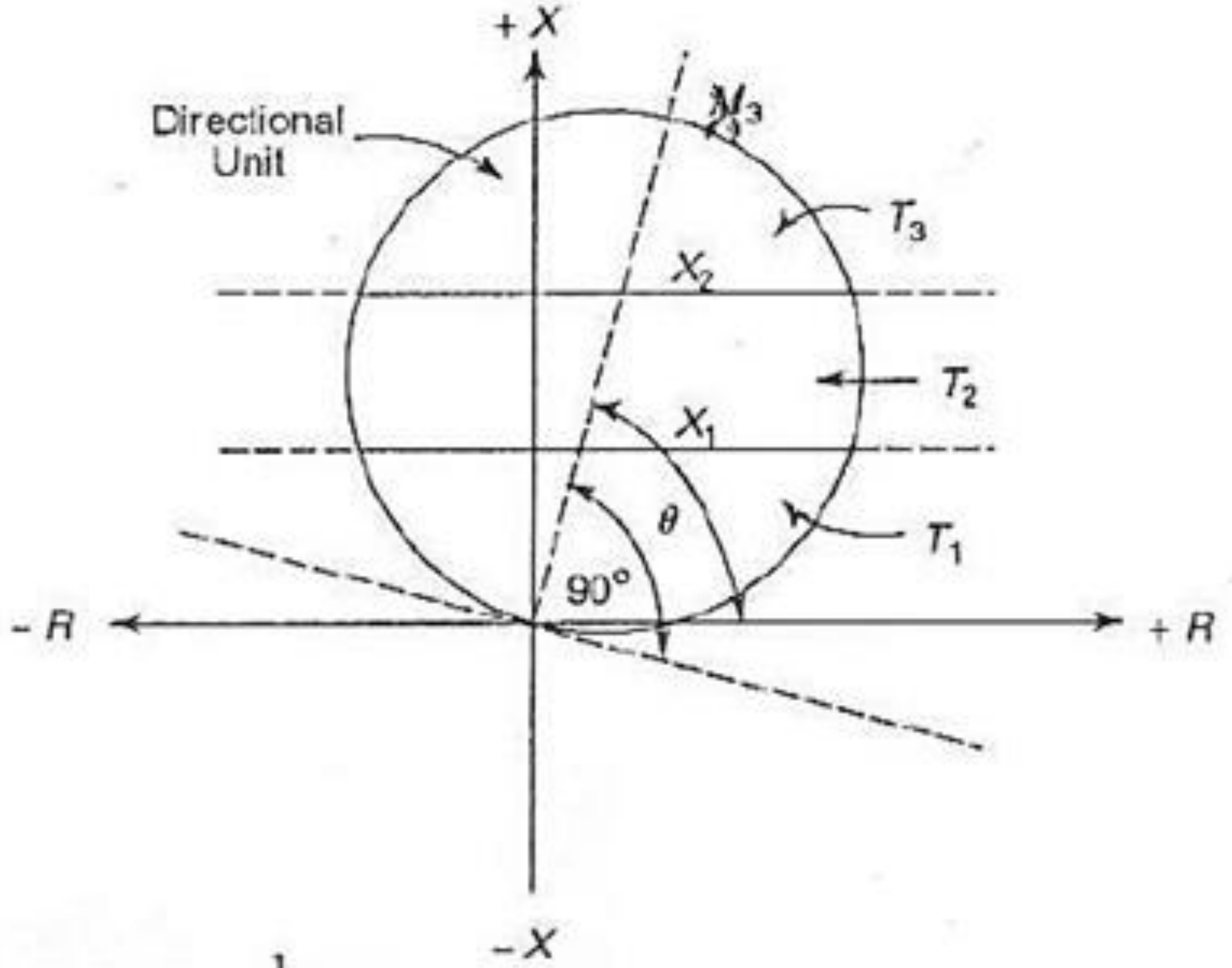


FIGURE 8.10 Reactance relay characteristics with directional unit

- As the reactance relay is a non-direction relaying unit, a direction relay I used in conjunction with it to provide direction features. The directional unit also serves the purpose of the third unit. The direction unit used for reactance relay has the characteristic of mho relay passing though the origin. The program of the direction unit is incorporated in the main program of the reactance relaying protective scheme. If the fault point lies within the protection zone of the direction unit then only the reactance relay program is taken up to check the position of the fault point, i.e. whether it lies in the I, II or III zone of protection. Depending upon the zone of protection, the tripping signal is sent with or without delay.

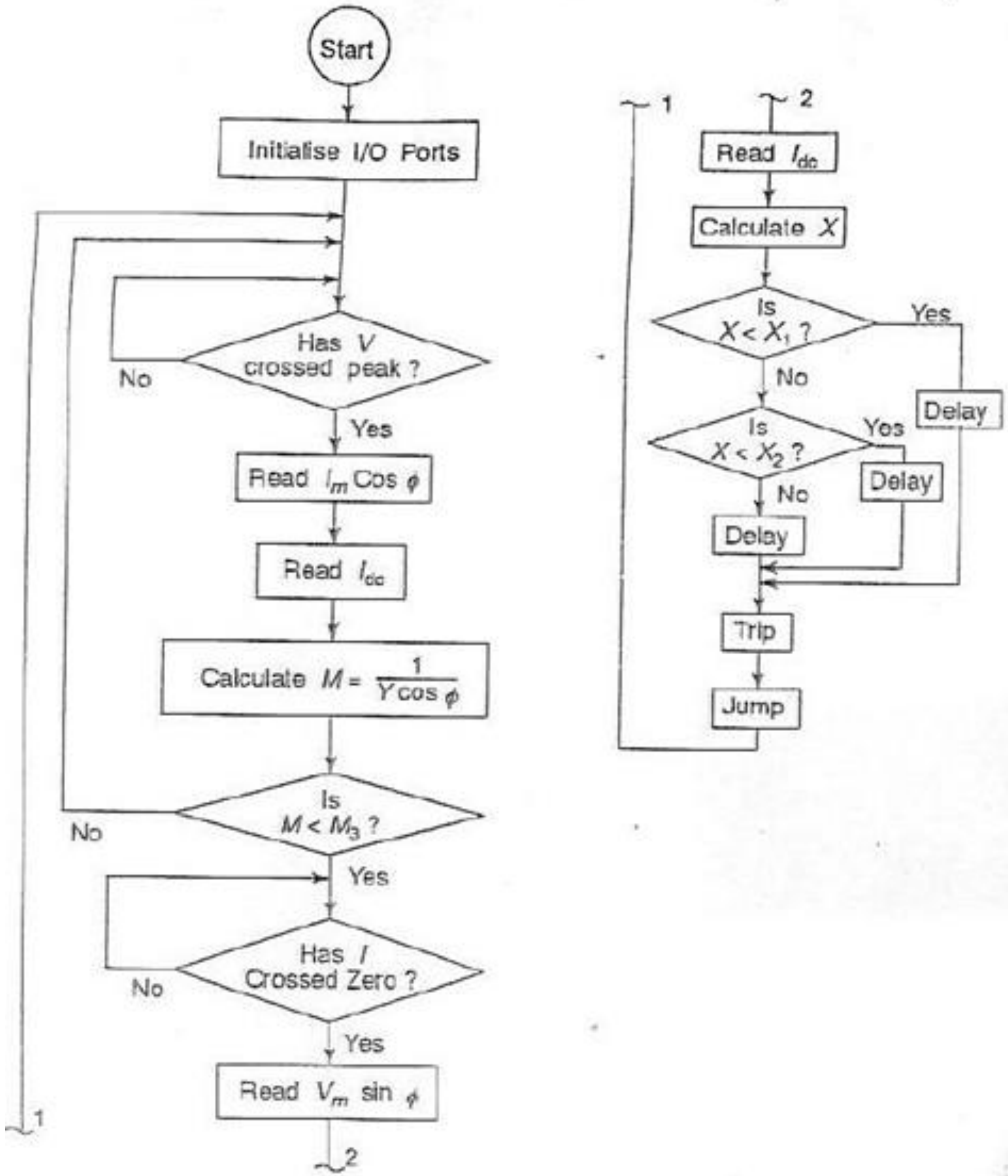
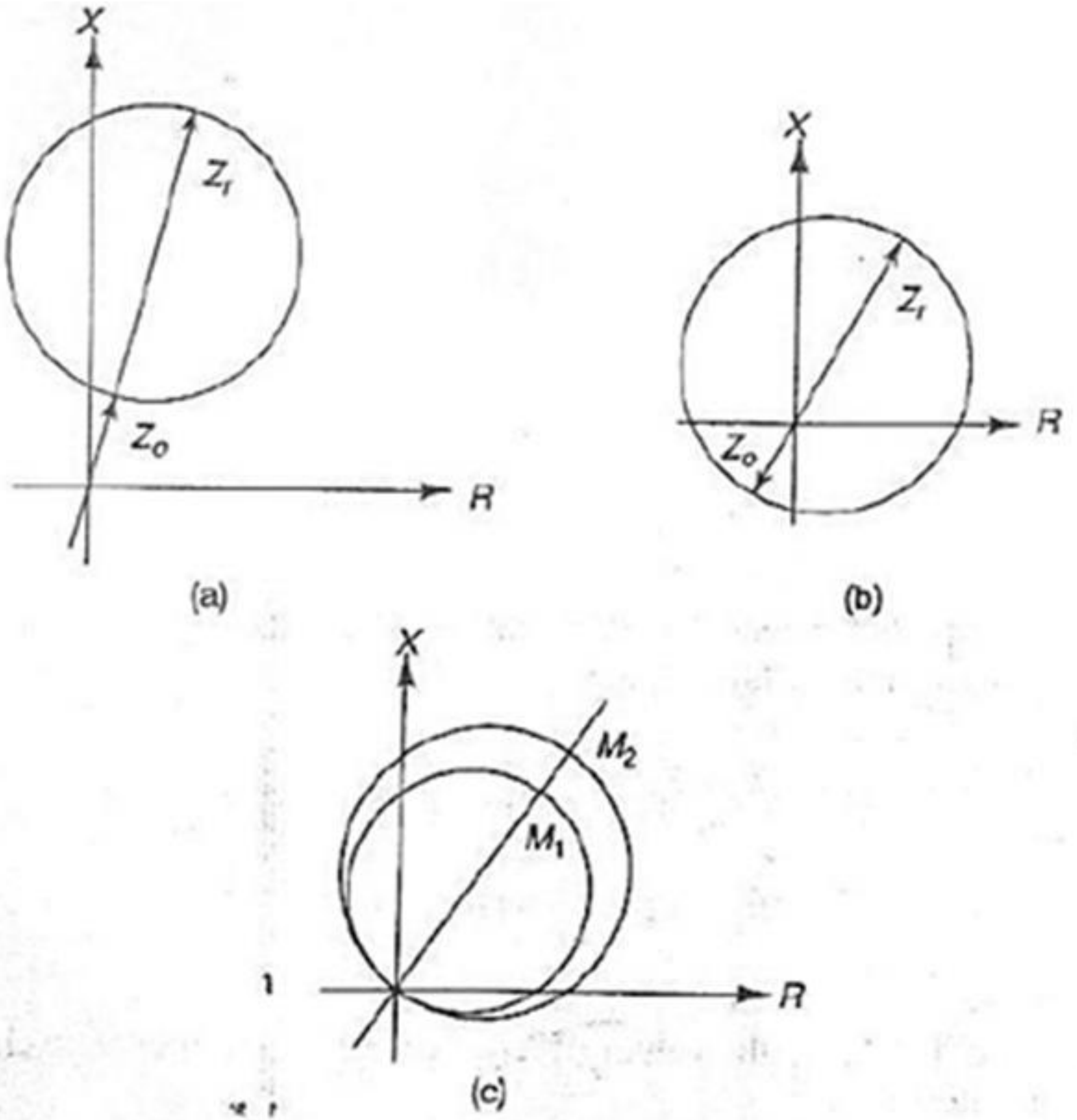


FIGURE 8.11 Program flowchart for reactance relay

Generalized Mathematical Expression for Distance Relays

- A generalized mathematical expression for the operation condition of mho, offset mho and impedance relay can be derived as follows. The direction of this expression relay can be derived as follows. The derivation of this expression is based on the operating condition of the offset mho relay having a positive offset.

  Figure 8.12 (a) shown the characteristic of an offset mho relay on the $R - X$ diagram.



E 8.12 (a) Offset mho relay with positive offset (b) Offset mho relay with negative offset (c) Mho relay

- The radius of this circle is

$$r = \frac{Z_r - Z_0}{2}$$

Where $Z_r = R_r + jX_r$ =impedance of the protected line section.

And $Z_0 = R_0 + jX_0$ = impedance by which the mho circle is offset

The center of the mho circle of offset from the origin by

$$C = \frac{Z_r + Z_0}{2}$$

if $Z(= R + jX)$ is the impedance seen by the relay, then the operating condition for the offset mho relay, shown in Fig.8.12(a),is given by

$$Z - C \leq r$$

Or $$\left| Z - \frac{Z_r + Z_o}{2} \right| \leq \left| \frac{Z_r - Z_o}{2} \right|$$

Or $$\left| R + jX - \frac{(R_r + jX_r) + (R_o + jX_o)}{2} \right| \leq \left| \frac{(R_r + jX_r) - (R_o + jX_o)}{2} \right|$$

Or $$\left| R - \frac{(R_r + R_o)}{2} + j(X - \frac{X_r + X_o}{2}) \right| \leq \left| \frac{(R_r - R_o) + j(X_r - X_o)}{2} \right|$$

$$\left[(R - \frac{(R_r + R_o)}{2})^2 + (X - \frac{X_r + X_o}{2})^2 \right] \leq \left[(\frac{(R_r - R_o)}{2})^2 + (\frac{(X_r - X_o)}{2})^2 \right]$$

R_r, R_0, X_r and X_o are constants for a particular characteristic and hence the above expression can be written in the generalised from as

$$\left[(R - K_1)^2 + (X - K_2)^2 \right] \leq K_3$$

- Where, the constants K_1, K_2 and K_3 are given by

$$K_1 = \frac{(R_r + R_o)}{2}$$

$$K_2 = \frac{(X_r + X_o)}{2}$$

$$K_3 = \left[\left(\frac{R_r - R_o}{2}\right)^2 + \left(\frac{X_r - X_o}{2}\right)^2 \right]$$

- Equation (8.7) is the generalized expression for the offset mho, mho and impedance relay. The value of K_1, K_2 and K_3 in the generalized expression are constant for a particular characteristic and different for different characteristics. Substituting the proper value of constants K_1, K_2 and K_3 the desired mho, offset mho or impedance characteristic can be realized. The value of K_1, K_2 and K_3 for different characteristic are computed as follows.

(i) for offset mho characteristic having negative offset as shown in Fig. 8.12(b), the negative value of R0 and X0 are used in computation of K1,K2 and K3 using equation (8.8).

$$K_1 = \frac{(R_r - R_o)}{2}$$

$$K_2 = \frac{(X_r - X_o)}{2}$$

and

$$K_3 = \left[\left(\frac{(R_r + R_o)}{2}\right)^2 + \left(\frac{(X_r + X_o)}{2}\right)^2 \right]$$

(ii) For mho characteristic as shown in Fig. 8.12(c), R0 and X0 are recreduced to zero, and consequenty the values of K_1, K_2 and K_3 obtained from Eq. (8.8) are as follows.

$$K_1 = \frac{R_r}{2}, \quad K_1 = \frac{R_r}{2} \quad \text{and} \quad K_3 = (\frac{R_r}{2})^2 + (\frac{X_r}{2})^2$$

(iii) For impedance chatacteristic, the displacement of the centre of the circle from the origin is zero.

i.c. $C = \frac{Z_o + Z_r}{2} = 0$ or $Z_o = -Z_r$

or $(R_o + jX_o) = -(R_r + jX_r)$

therefore, $R_o = -R_r$ and $X_o = -X_r$

then from Eq.(8.8), the values of K_1, K_2 and K_3 for impedance characteristic are as follows.

$$K_1 = K_2 = 0 \qquad \text{and} \qquad K_3 = R_r^{\,2} + X_r^{\,2}$$

Consequently, the operating condition for the impedance relay becomes

$$R^2 + X^2 \leq R_r^2 + X_r^2$$

Or $R^2 + X^2 \leq r^2$, where r is the radius of the circle. Therefore, a offset mho, mho or impedance characteristic can easily be realized by using the generalized Eq. (8.7) and substituting the appropriate values of K_1, K_2 and K_3 for the desired characteristic.

Measurement of R and X

- In microprocessor-based distance relaying, the microprocessor calculates the active and reactive components (R and X) of the apparent impedance (Z) of the line from the relay location to the fault point from the ratios of the appropriate voltages and currents, and then compares the calculated values of R and X to the pickup value of the relay to be realized in order to determine whether the fault occurs within the protective zone of the relay or not. The active and reactive components of the apparent impedance (Z) are resistance (R) and reactance (X), respectively. The algorithm presented in this section for the calculate of R and X assumes that the waveforms of voltage and current presented to the relay are pure fundamental frequency sinusoids. But the post-fault voltage and current waveforms fail to be pure fundamental frequency sinusoids due to the presence of harmonics and dc offset components signals is necessary in order to eliminate harmonics and dc offset components and to obtain pure fundamental frequency sinusoidal signals. An active bandpass filter having a band-pass of 40 to 60 Hz is employed to filter harmonics and dc offset. Filtering of harmonics and dc offset from the post-fault voltage and current signals can be achieved by employing digital filters. A number of digital algorithms, based on the digital filtering technique have been proposed for the calculation of R and X.

Measurement of resistance

- The resistance as seen by the relay from the relay location to fault point is given by:

$$R = Z\cos\Phi = \frac{V}{I}\cos\Phi$$

$$= \frac{V_m\cos\Phi}{\sqrt{2}A_1 I_{dc}} \quad \text{(as } I \propto I_{dc} \text{ or } I = A_1 I_{dc} \text{ where } A_1 \text{ is a constant.)}$$

$$= A\frac{V_m\cos\Phi}{I_{dc}}$$

Where A is a constant, and equal to $1/\sqrt{2}A_1$.

- $V_m \cos\Phi$ is the instantaneous value of the voltage at the moment of peak current, as shown in Fig. 8.13 (a). Therefore, the resistance is proportional to the ratio of voltage at the instant of current peak to the rectified current I_{dc}. To obtain a pulse at the moment of peak current, a phase shifting circuit and a zero-crossing detector have been used. The current signal is fed to the phase shifter to get a phase shifter of 90°. Then the output of the phase shifter is fed to the zero crossing detector, as shown in Fig. 8.13 (b) to obtain the required pulse. The microcomputer reads the output of the zero-crossing detector, in order to examine whether the current signal has reached its peak. As soon as the current crosses its peak, the microcomputer sends a commend to the multiplexer to switch on channel S_4 to obtain the instantaneous value of the Voltage at the moment of peak current, which is equal to $V_m \cos\Phi$. The microcomputer reads this instantaneous value of the voltage through the A/D converter, and stores the digital voltage in the memory. The current signal is converted to dc using a precision rectifier. The microcomputer gets the rectified current I_{dc} through the multiplexer channel S_7 and A/D converter. After getting the value of $V_m \cos\Phi$ and I_{dc}, the microcomputer calculates the value of resistance.

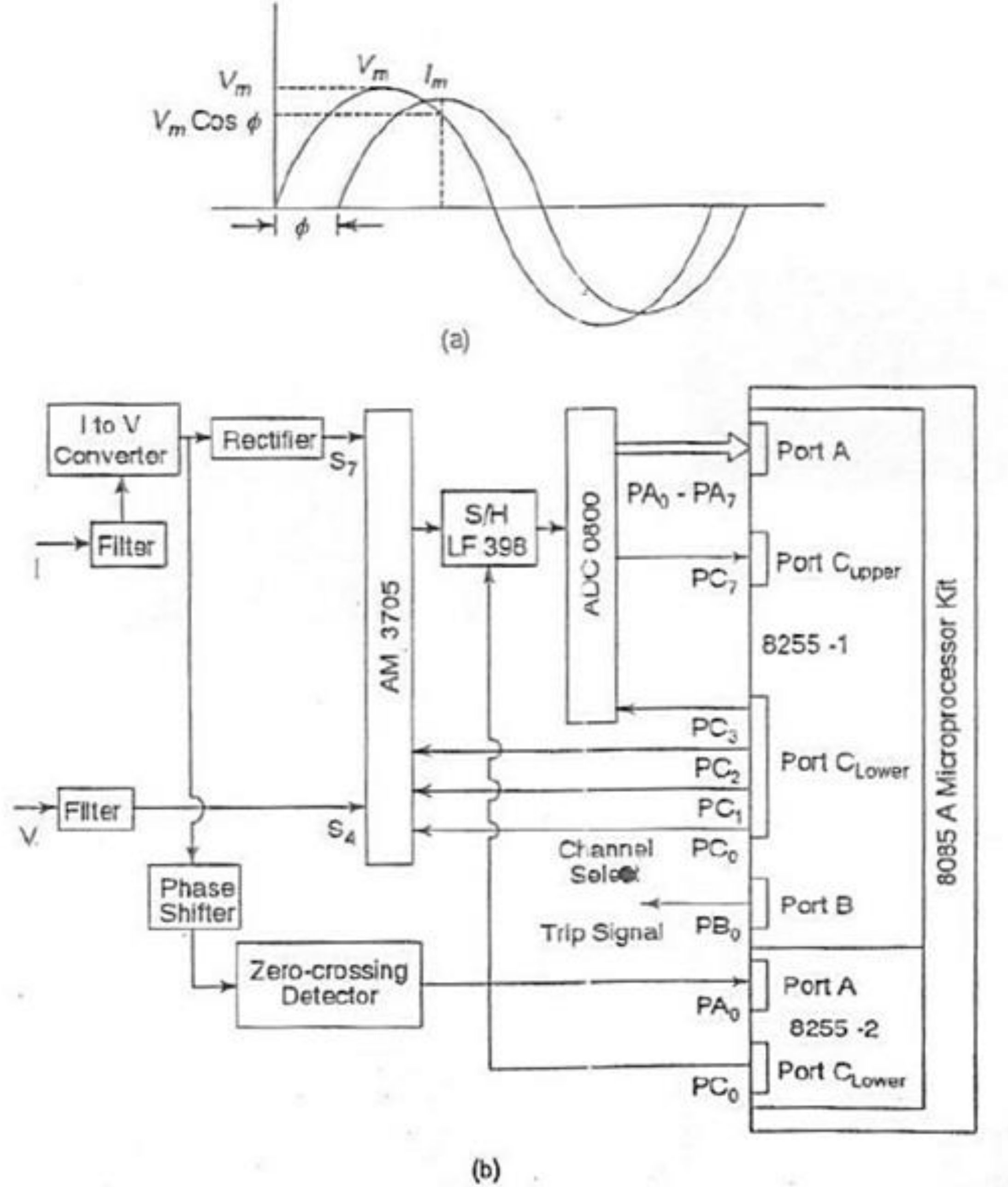


FIGURE 8.13 (a) Instantaneous value of voltage at the instant of peak current
(b) Block schematic diagram of interface for resistance measurement

Measurement of reactance

- The reactance seen by the relay is given by

$$X = Z \sin \Phi = \frac{V}{I} \sin \Phi$$

$$= \frac{V_m \sin \Phi}{\sqrt{2} A_1 I_{dc}}$$

$$= A \frac{V_m \sin \Phi}{I_{dc}}$$

- The instantaneous value of the voltage at the moment of zero current is $V_m \sin\Phi$, as shown in Fig. 8.8. Therefore, the reactance is proportional to the ratio of voltage at the moment of current zero to the rectified current. The measurement of reactance using this technique has already been discussed in section 8.5, while describing the reactance relay. The interface for the measurement of reactance will be the same as that for the reactance relay, i.c. as shown in Fig. 8.9. The microcomputer reads the output of the zero-crossing detector and examines whether the current has reached its zero instant. As soon as the current crosses its zero, the microcomputer reads the instantaneous value of the voltage through the multiplexer channel S_4 and A/D converter. Then the microcomputer reads the rectified current I_{dc} through the multiplexer channel S 7 and A/D converter. After receiving the values of $V_m \sin\Phi$ and I_{dc} the microcomputer computes the reactance X. the program flowchart for the measurement of R and X is shown in Fig. 8.14.

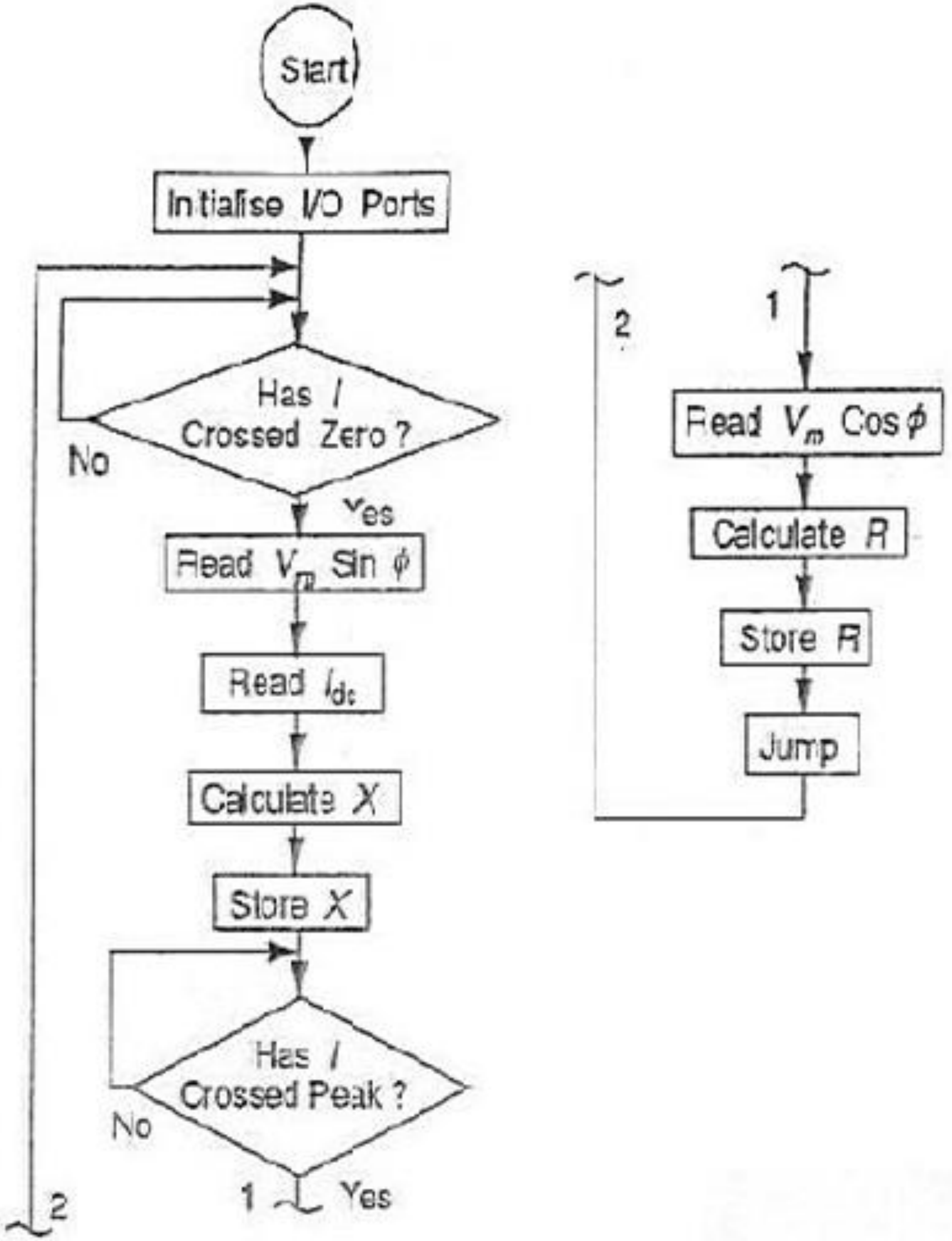


FIGURE 8.14 Program flowchart for measurement of R and X

Mho and offset Mho relays

- The characteristic of a mho relay on the impedance (R-X) diagram is circle passing through the origin, as shown in Fig. 8.12(c). With such a characteristic, the mho relay is inherently directional as it detects fault in the forward direction only the mho characteristic, occupying the least area on the R-X diagram is least affected by power surges which remain for longer periods in case of long lines. Therefore, a mho relay is best suited for the protection of long transmission lines. Therefore, a mho relay is best suited for the protection of long transmission lines against phase faults though this relay is affected by are resistances more than any other type of the relay, the value of are resistance in comparison to the impedance of the long lines is much less. Thus, an arcresistance does not affect much the performance of a mho relay in case of long lines, for very long lines, the tripping area of mho relays can be futher reduced by blinders.

- Offset mho relays, whose characteristics are shown in Fig. 8.12(a) and (b) are used for zone III in a three –zone protective scheme employing mho relays for zone I and II for the protection of power transmission lines against phase faults. This scheme is shown in Fig. 8.15(a) for long transmission lines, mho and offset mho characteristics, as shown in Fig. 8.15(b) are used to discriminate between loads and fault. An offset mho characteristic with negative offset has more tolerance to are resistance. By offsetting the III zone mho characteristic to overlap the origin, it can also be used for power awing blocking.
- The tripping area of a mho characteristic can be restricted by two overlapping mho characteristics as shown in Fig. 8.16(a) and (b). The tripping area is the common area between two mho characteristics. This type of characteristic occupies a very small area on the R-X diagram and hence it is least affected by power surges. A restricted mho characteristic can be used for the protection of very long lines. To take care of the resistive faults near the busbar, offset mho characteristic, as shown in Fig. 8.16(b) can be used.

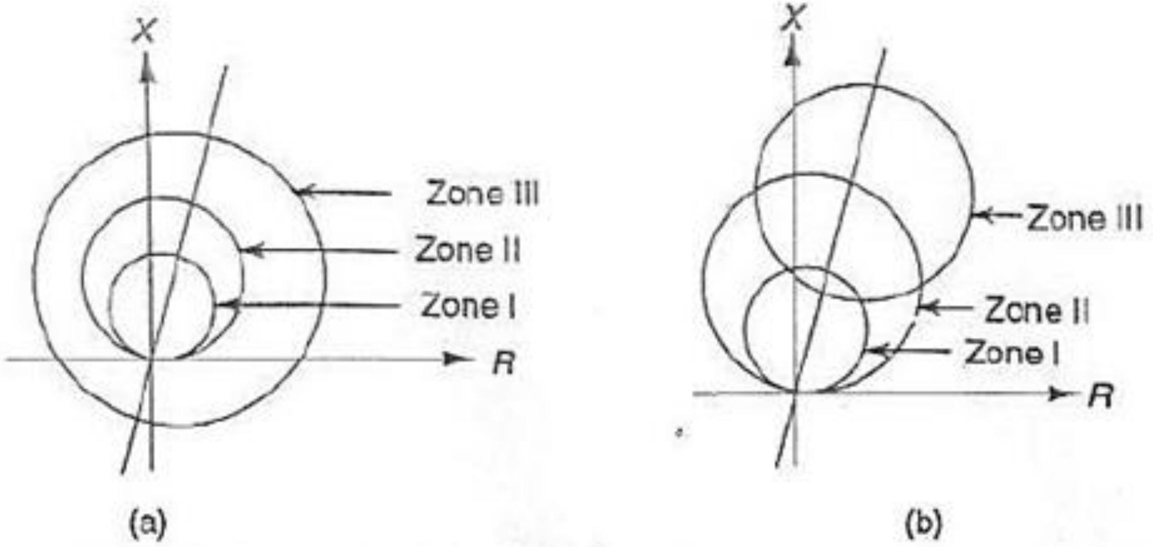
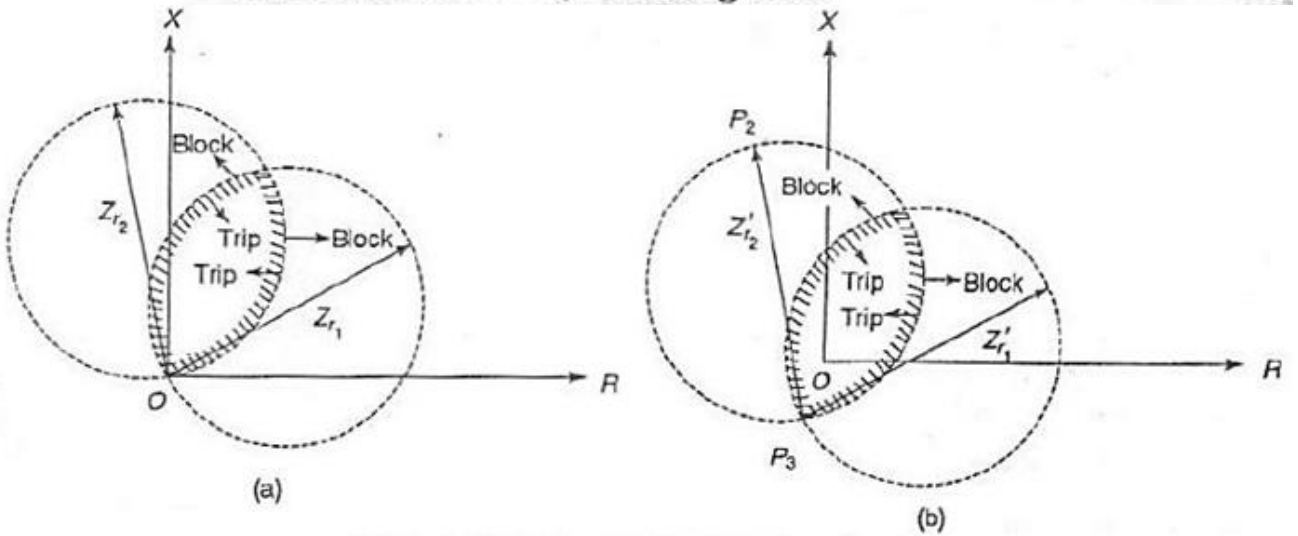


FIGURE 8.15 (a) Mho and offset mho relays for transmission lines (b) Mho and offset Mho relays for long lines



16 (a) Restricted mho passing through origin
(b) Réstricted offset mho relay

Realization of Mho Characteristic

- The characteristic of a mho relay can be realized using three techniques i.e.

I. instantaneous amplitude comparison technique

II. phase comparison technique

III. generalized mathematical expression.

(i) Instantaneous Amplitude comparison Technique.

- The characteristic of a mho relay is realized by comparing the instantaneous value of current at the moment of voltage peak against the rectified voltage. The instantaneous value of current at the moment of voltage peak is $I_m \cos\Phi$ as shown in Fig.8.5(b).the condition to be satisfied for the operation of the relay is as follows.

$$I_m \cos\Phi > K_1 V_{dc} \quad \text{or} \quad \frac{I_m}{V_{dc}} \cos\Phi > K_1$$

or $$\frac{I}{V}\cos\Phi > K_2$$ (as I_m and V_{dc} are proportional to the rms values of I and v respectively)

or $$Y\cos\Phi > K_2 \quad \text{or} \quad \frac{1}{Y\cos\Phi} < \frac{1}{K_2}$$

or $$M < K$$

Where K_1, K_2 and K are constants.

- If the design angle θ is introduced while feeding the voltage and current signals to the relay, the above expression is modified and is given by

$$\frac{1}{Y\cos(\Phi-\theta)} < K$$

By changing θ a mho characteristic can be shifted towards the R-axis to increase its tolerance to arc resistance, as shown in Fig. 8.17. The value of θ is not keep more than 75° to have a reasonable tolerance for arc resistances when a fault occurs near the bus.

- The block schematic diagram of the interface for realization of mho relay is shown in Fig. 8.18. The voltage signal is fed to the phase-shifter to get a phase-shift of 90°. Then the output of the phase shifter is fed to the zero-crossing detector to get the required pulse. The microcomputer reads the output of the zero-crossing detector and cheeks whether the voltage has crossed its peak value. After getting the pulse at the instant of voltage peak, the microcomputer sends a command to the multiplexer to switch on the channel S_2 to get the instantaneous value of the current at the moment of peak voltage. This instantaneous value of the current which is equal to $I_m \cos\Phi$ is fed to an A/D converter. After the conversion is over, the digital output of the converter is stored in the memory. The microcomputer then gets the rectified voltage V_{dc} through the multiplexer channel S_5 and the A/D converter. This quantity is also stored in the memory. After obtaining this data the microcomputer computes $V/I\cos\Phi$ which is proportional to $V_{dc}/I_m\cos\Phi$ and is equal to M. the calculated value of M is compared whit the predetermined value of M_1 which remains stored in the memory. M_1, M_2 and M_3 are the predetermined value of M for I, II and III zones of protection, respectively. If M is less than M_1, the tripping signal is sent instantaneously. If M is greater than M_1 but less than M_2, the tripping signal is sent after a predetermined delay. If M is greater than M_2 but less than M_3, a greater delay is provided to send the tripping signal. If M is greater than M_3, the microcomputer goes back to the starting point, starts reading the voltage and current signals and again proceeds according to the program. The program flowchart is shown in Fig. 8.19.

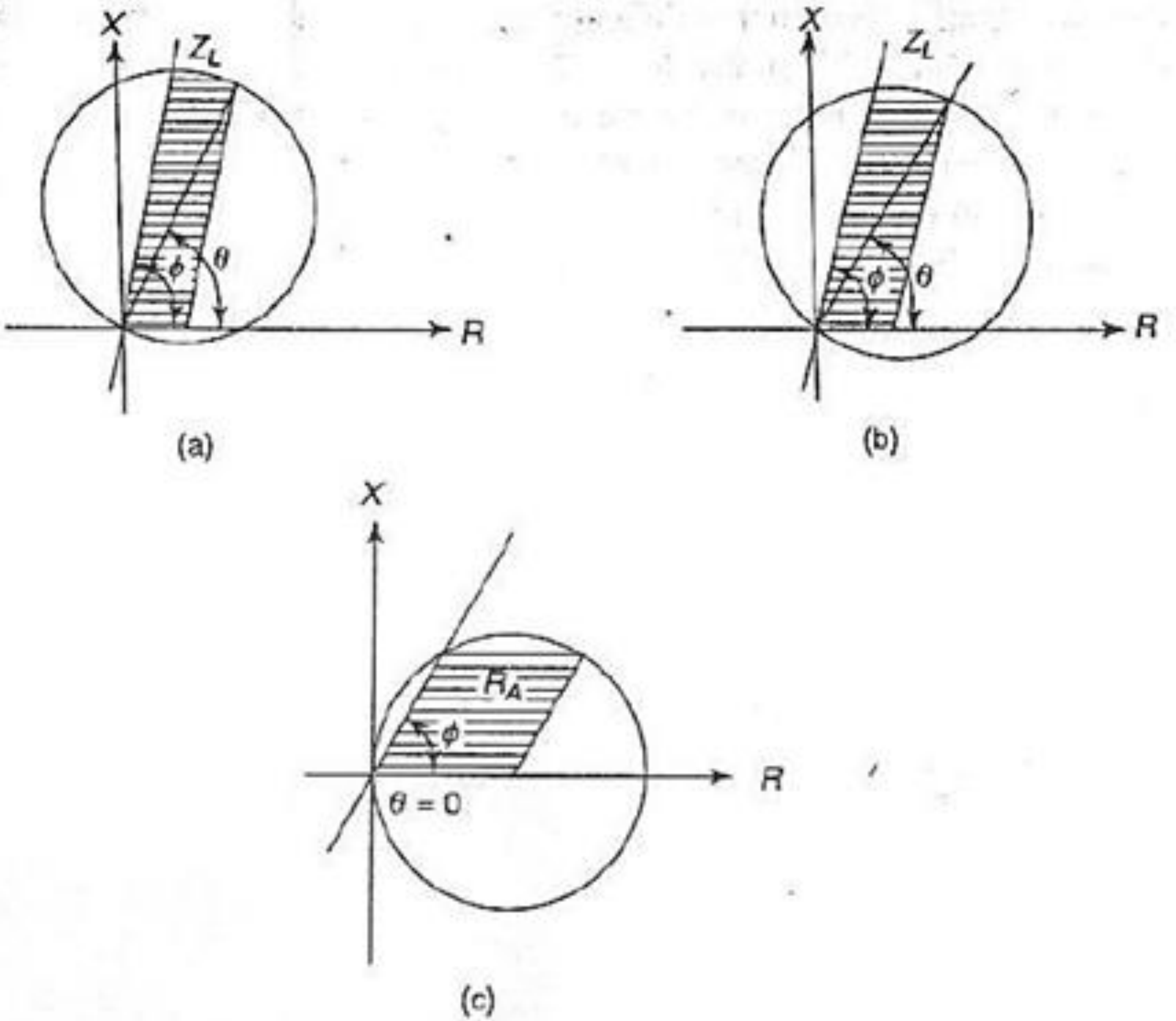


FIGURE 8.17 Mho relay's characteristics with different values of θ

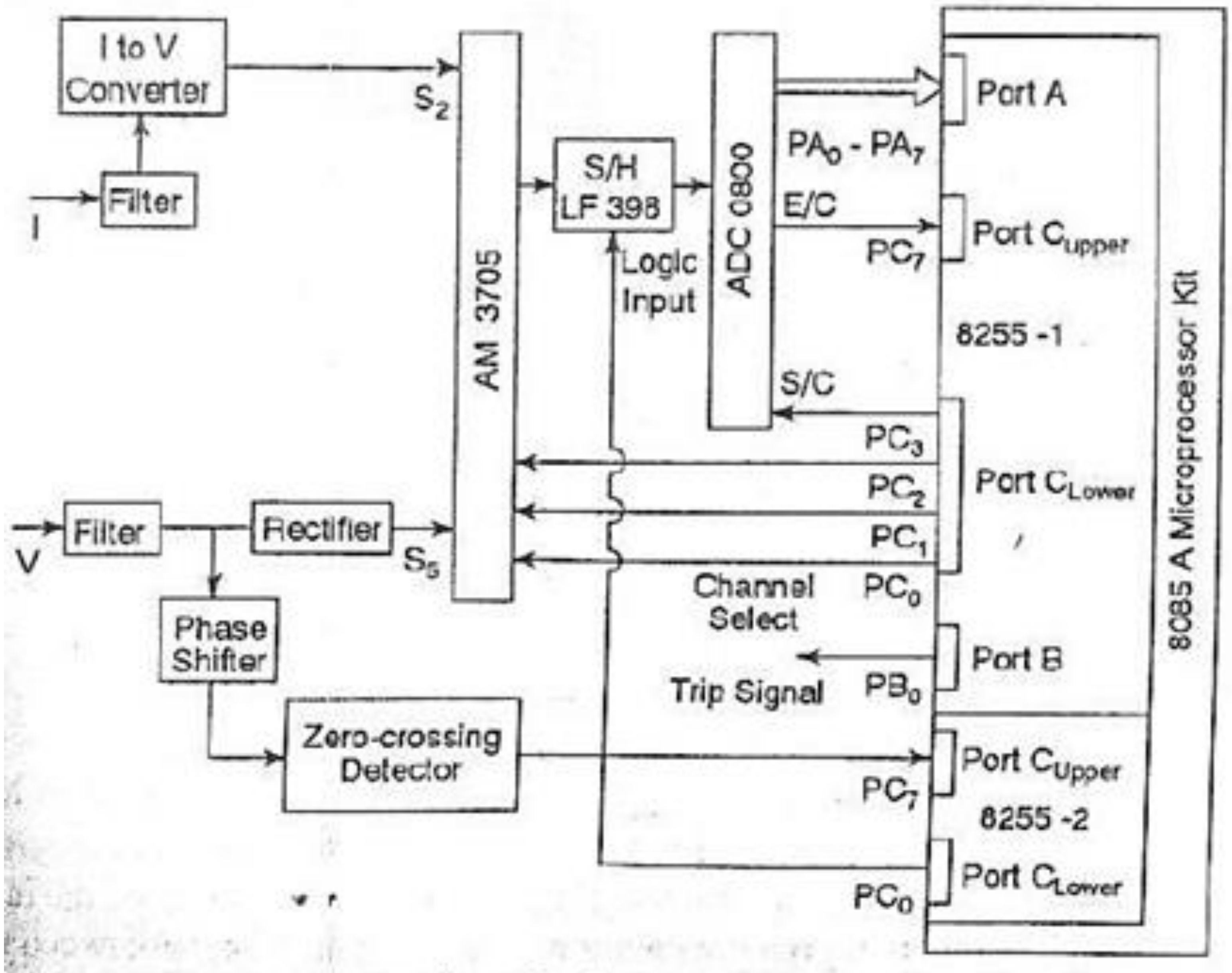


FIGURE 8.18 Block schematic diagram of interface for mho relay

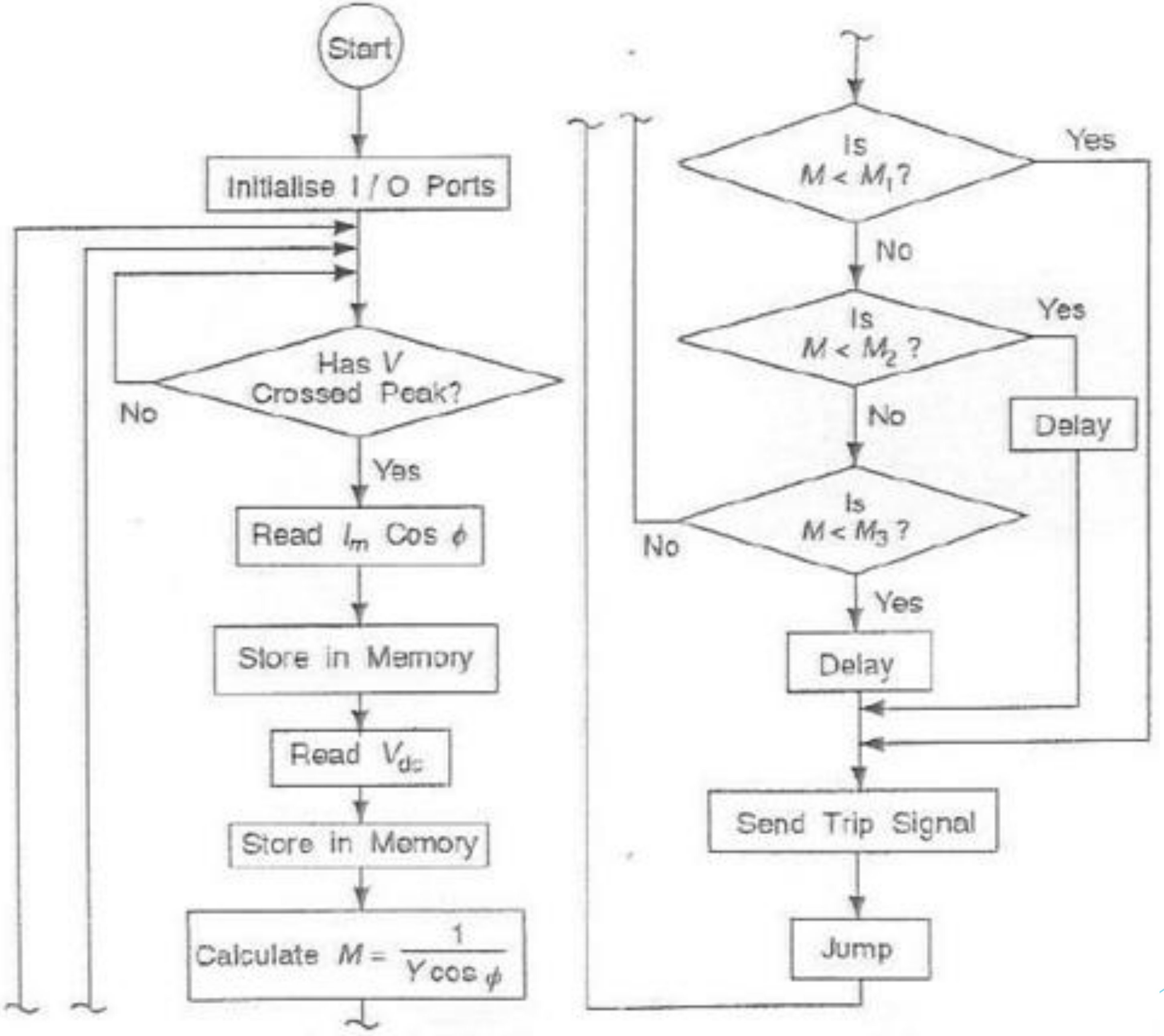
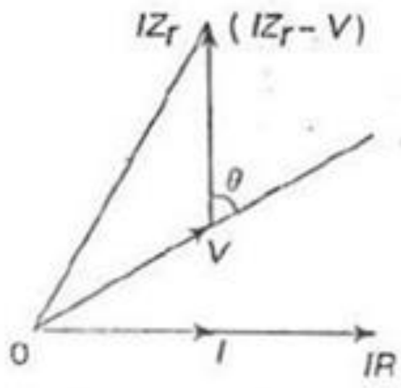


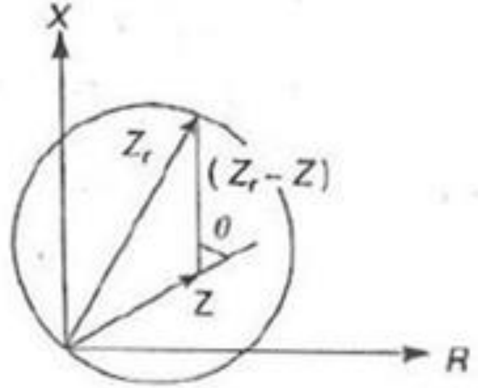
FIGURE 8.19 Program flowchart for mho relay

(ii) phase Comparison Technique

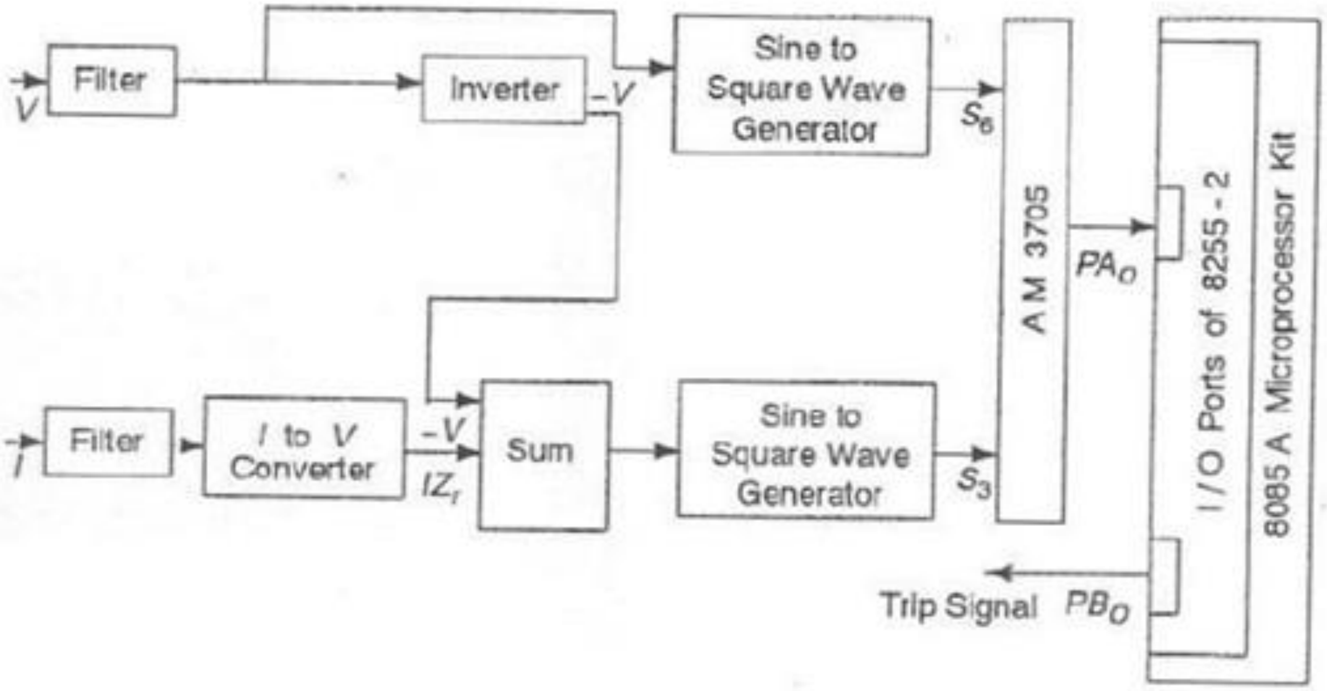
- For realization of a mho characteristic, the phase angle between ($IZ_r - V$) and V is measured. Figure 8.20(a) showns a phasor diagram showing V , IR and IZ_r . The phasors of this diagram are divided by I and the resulting phasor diagram is obtained in the Z-plane as shown in Fig. 8.20(b). The diameter of the mho circle is Z_r . For any point lying within the circle, the phase angle between the phasors ($Z_r - Z$) and Z is less than $\pm 90^\circ$. For points lying on the right hand side of the diameter, the phase angle is positive and for points lying on the left hand side, it is negative.
- The block schematic diagram of the interface for the measurement of phase angle between ($IZ_r - V$) and V is shown in Fig. 8.20(c). The ac input signal are converted in to square waves. The phase angle is measured between the positive going zeros of the square wanes. The phasor ($IZ_r - V$) is taken as reference. For measurement of phase angle between ($IZ_r - V$) and V , the microcomputer measures the time between positive going zero points of their corresponding square waves. The phase angle is proportional to the measured time. The microcomputer then compares the measured phase angle with 90° , in order to determine whether the fault point lies within the mho circle or not. If the measured phase angle is less than 90° , the microcomputer issues a tripping signal.



(a) Phasor diagram



(b) Phasor diagram in Z-plane



(c) Schematic diagram of interface

FIGURE 8.20 Mho relay

(iii) Generalised Expression Technique

- The mho characteristic realized with the help of a generalized expression (8.7). For characteristic, the value of constants K_1, K_2 and K_3 are respectively equal to $R_r/2, X_r/2$ and $(R_r/2)^2 + (X_r/2)^2$. The constants are predetermined and stored in the memory. The microcomputer first of all calculates the resistance and reactance (R and X) as seen by the relay, and then examines whether the operating condition as expressed by Eq. (8.7) is satisfied or not. If this condition is satisfied, the microcomputer sends a tripping signal. The block diagram of the interface is shown in Fig. 8.21.

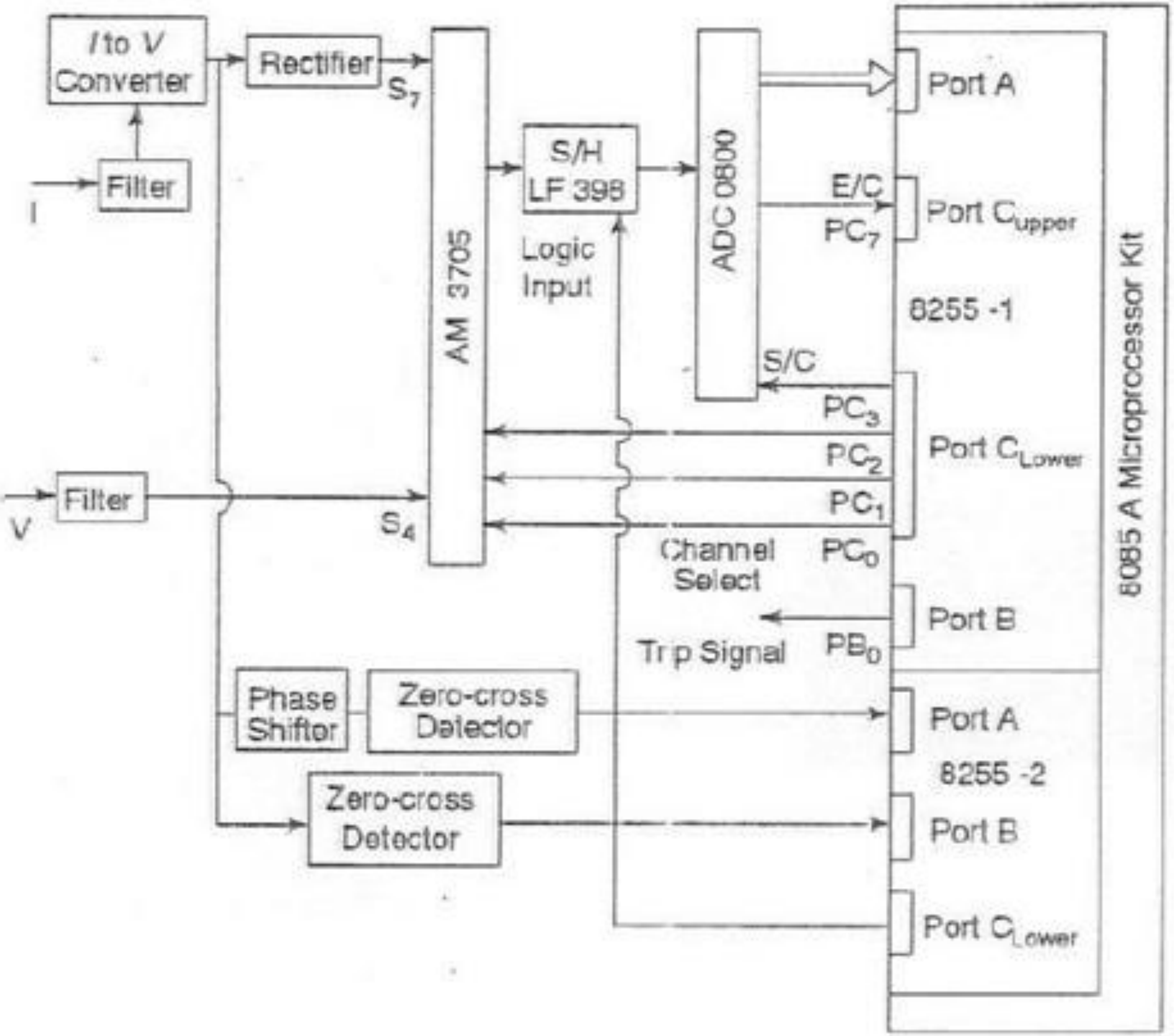


FIGURE 8.21 Block schematic diagram of interface for mho, offset mho, restricted mho and quadrilateral relays

Realization of Offset Mho Characteristic

- The offset mho characteristic is also realized using the generalized expression or Eq. (8.7). The constants K_1, K_2 and K_3 for the offset mho characteristic are predetermined and stored in the memory. The microcomputer measures R and X at the relay location, makes calculations for different terms of the equation (8.7), and then checks whether the operating condition is satisfied. If the operating condition is satisfied, the microcomputer sends the tripe signal. The block diagram of the interface is shown in Fig. 8.12 and the program flowchart in Fig. 8.22.

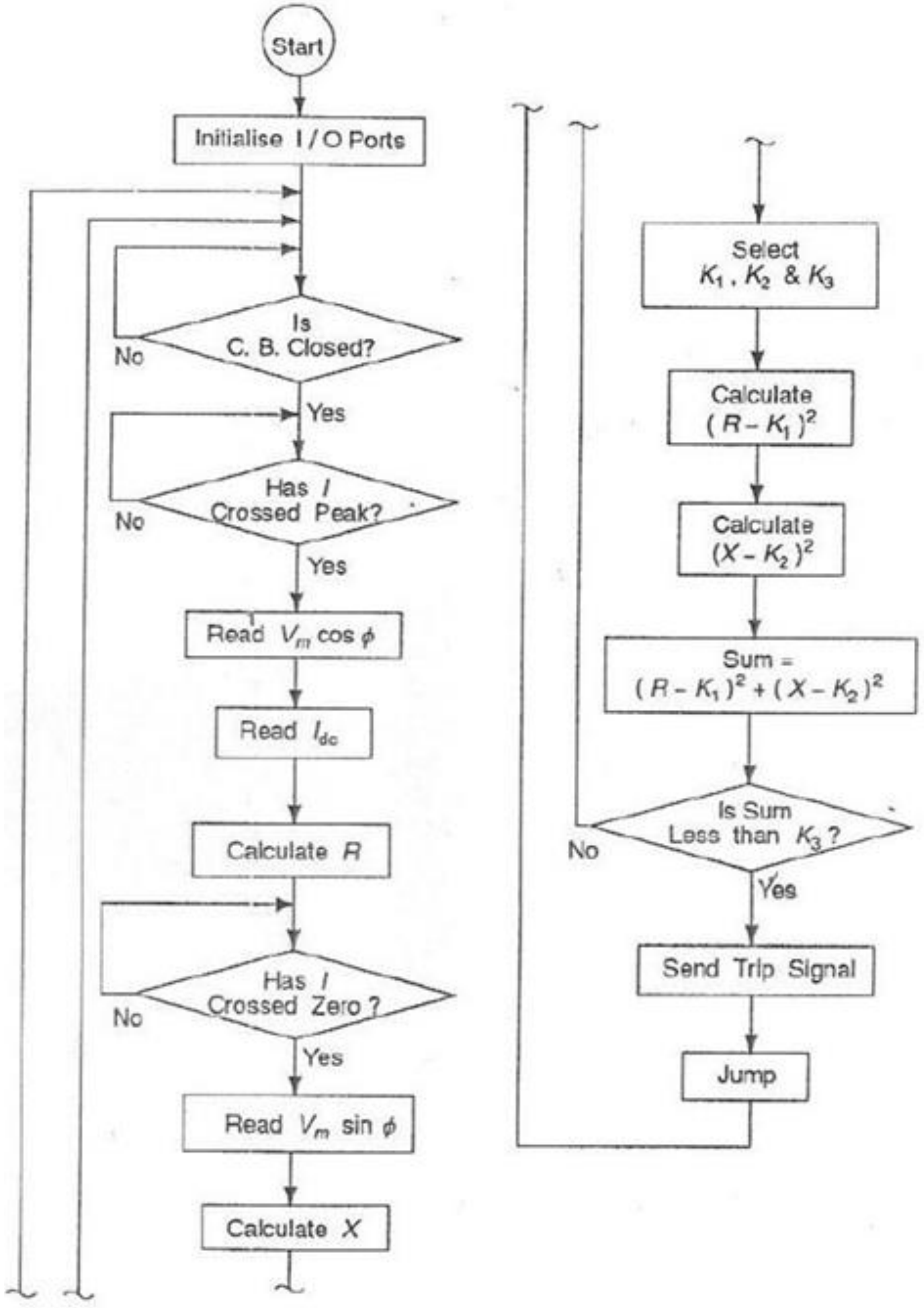


FIGURE 8.22 Program flowchart for offset mho relay

Realization of restricted Mho Characteristic

- Restricted mho and restricted mho offset mho characteristics are shown in Fig. 8.16 (a) and (b). the block schematic diagram of interface for realization of these characteristics is shown in Fig.8.21. For these characteristic, the mho or offset mho circle, which is near the X-axis has a negative value for the constant K_1. Therefore, the term $(R+K_1)^2$ replaces term $(R-K_1)^2$ in the expression of Eq. (8.7). The microcomputer first of all measures R and X and then examines whether the fault point lies in the mho characteristic which is near the R-axis. If the fault point does not lie in this circle, the microcomputer then checks whether it also lies within the II circle which is near the X-axis. If the fault point lies within both circle, i.e. in the common area of the circle, the microcomputer sends a tripping signal to the circuit breaker. The program flowchart is shown in Fig. 8.23.

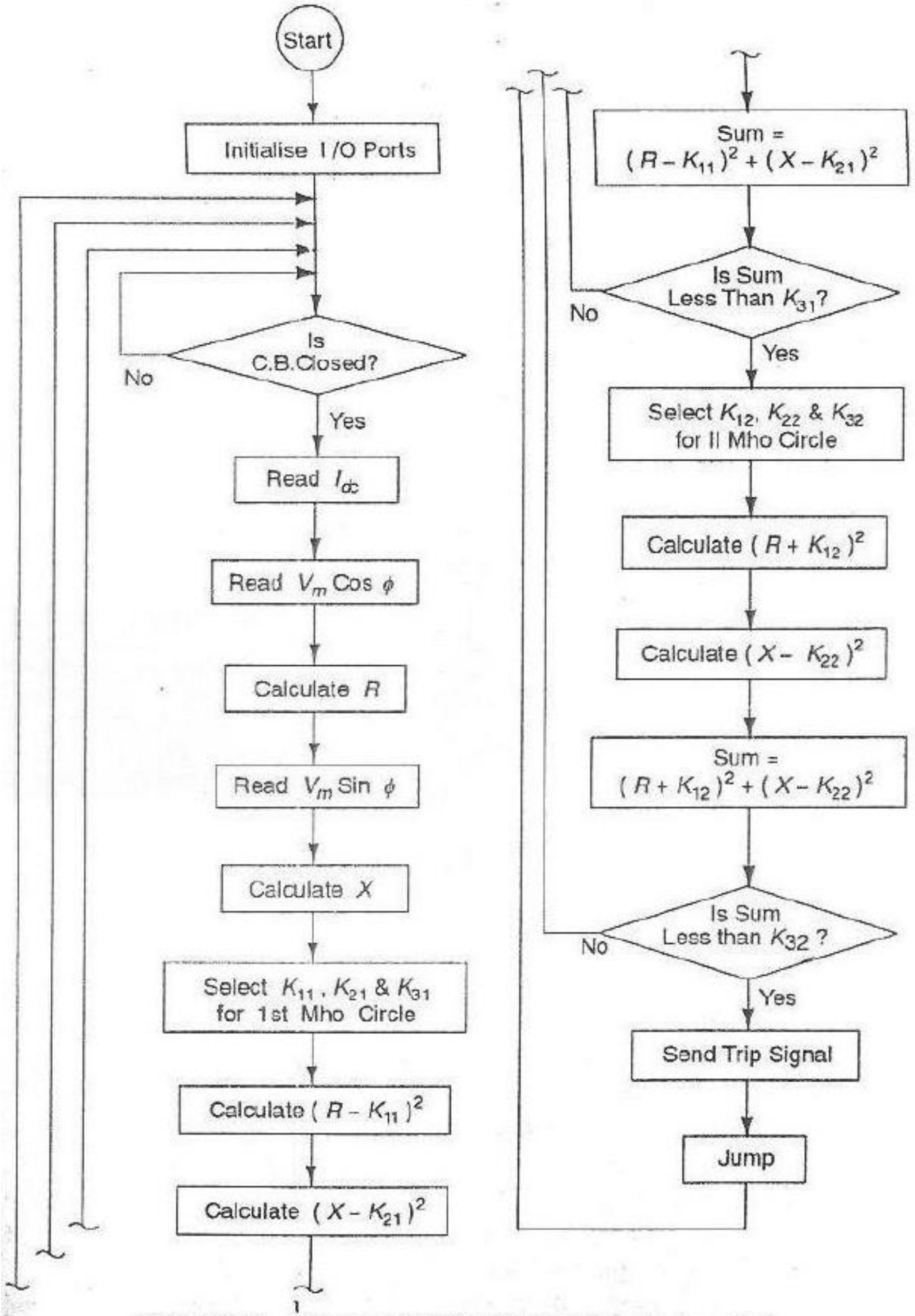


FIGURE 8.23 Program flowchart for restricted mho relay

Quadrilateral Relay

- The quadrilateral relay is best suited for the protection of EHV/UHV and ELD transmission lines as it possesses the valuable property of possessing the least tendency for mal-operation due to heavy power swings, fault resistance and overloads. It is also suitable for short and medium lines. Its characteristic can be designed to just enclose the fault area of the line to be protected. The electromagnetic version of this relay requires four units, each corresponding to one side of the quadrilateral, or a mho relay with two blinders. A static relay employing a multi-input comparator gives a better quadrilateral characteristic than the electromagnetic relays. But this relay does not possess the flexibility needed for obtaining different characteristic. A microprocessor-based scheme can easily obtain a quadrilateral characteristic using the same interface which is used for other types of distance relay. It has the flexibility to obtain a desired quadrilateral characteristics by simply providing the proper data. Quadrilateral characteristics are shown in Fig. 8.24 (a) and (b).

- The block schematic diagram of the interface for realisation of quadrilateral characteristic is shown in Fig. 8.21. The microcomputer measures the resistance and reactance at the relay location using the technique as described in Section 8.7.
- The measured value of the resistance and reactance are compared with the predetermined values of resistance and reactance, respectively. The predetermined values of resistance and reactance are stored in a tabular from in the memory. These values are selected to give the desired quadrilateral characteristic. The characteristic can easily be extended in the resistive direction using appropriate data in the memory to cater for large fault resistance in the case of a short line. The extension of characteristic in the resistive direction is independent the reactive direction.
- Corresponding to a particular value of the line reactance, R_l and R_h are the lower and higher limits of resistance on the characteristic curve shown in Fig. 8.24 (a). To obtain the desired quadrilateral and stored in the memory. The microcomputer first measures the line reactance X and compares it with X_3 the predetermined value of the reactance for the third zone of protection. If X is greater than X_3, it goes back to the starting point and measures the same again.

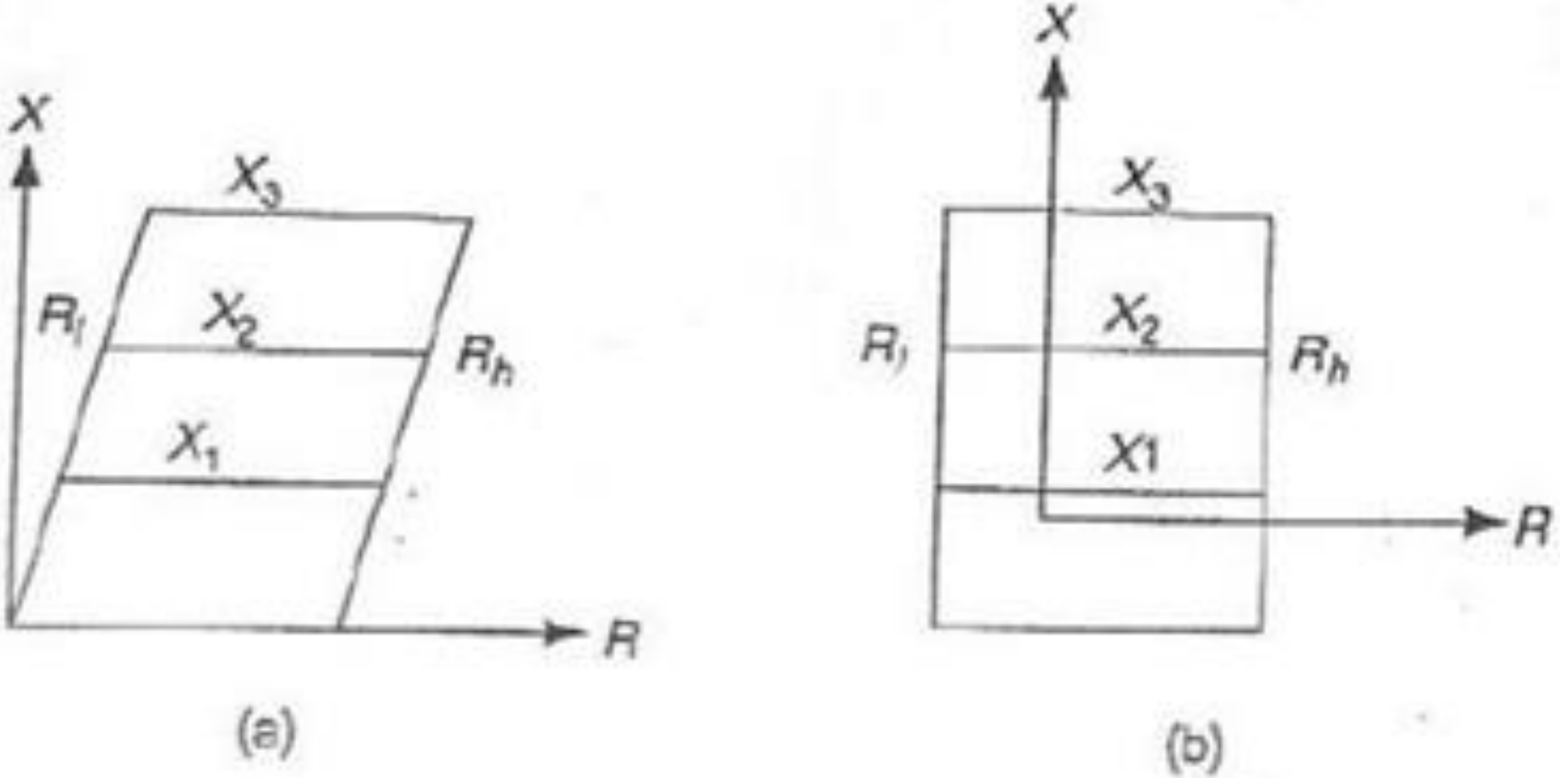
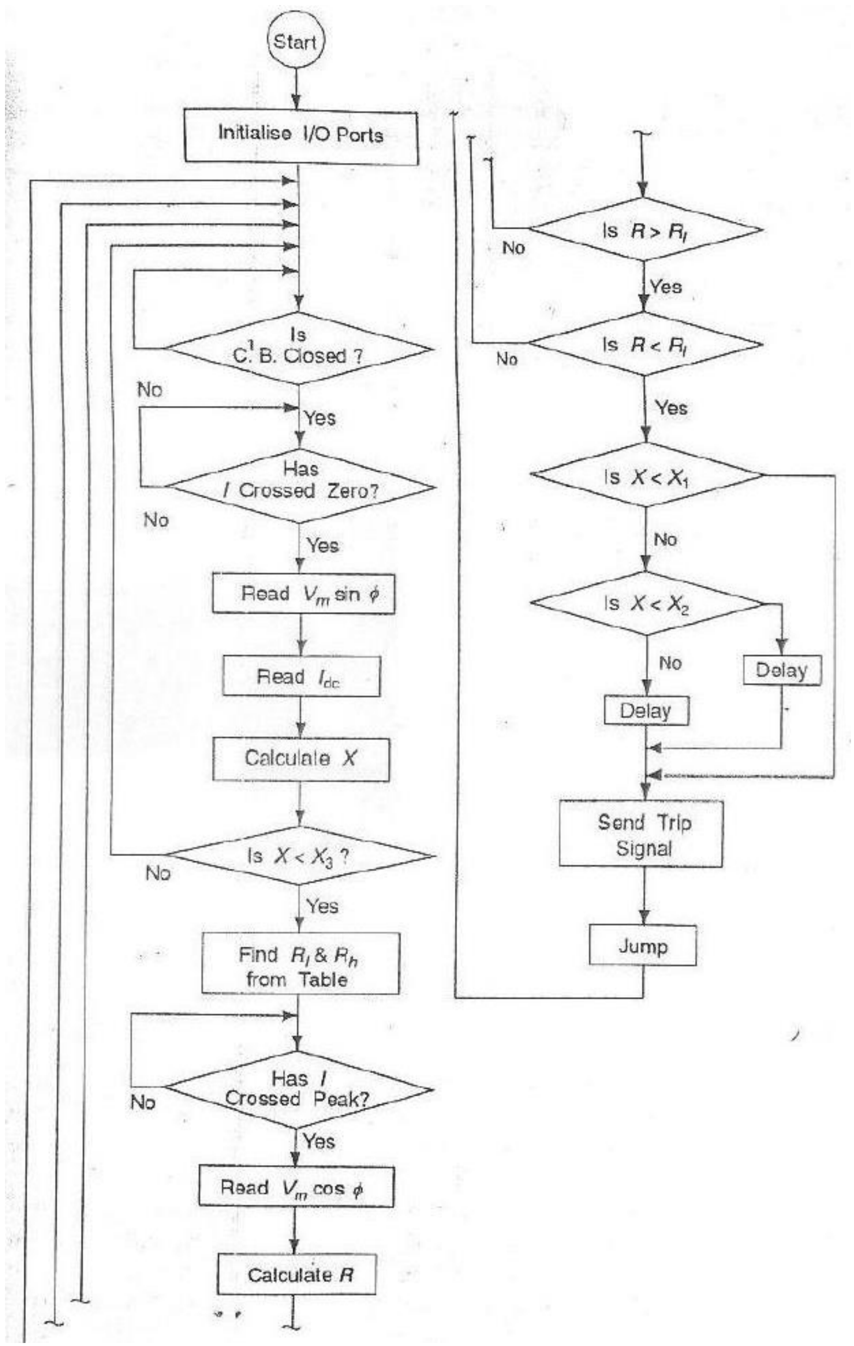


FIGURE 8.24 (a) Quadrilateral relay passing through origin
(b) Offset quadrilateral relay

- If X is less than X_3, it proceeds further to measure R, the resistance seen by the relay. Corresponding to X, the measure values of the reactance, the microcomputer selects the values of R_l and R_h from the look-up table. For the operation of the relay, the condition to be satisfied is $R_l < R < R_h$. If X is less than X_1, a tripping signal is sent to the circuit breaker instantaneously. If X is greater than X1 but less than X_2, the tripping signal is sent after a predetermined delay. If X is greater than X_2 but less than X_3, a greater delay is provided. The program flowchart is shown in Fig. 8.25.


Start
Initialise I/O Ports
Is C.B. Closed ?
No
Yes
Has I Crossed Zero?
No
Yes
Read $V_m \sin \phi$
Read I_{dc}
Calculate X
Is $X < X_3$?
No
Yes
Find R_l & R_h from Table
Has I Crossed Peak?
No
Yes
Read $V_m \cos \phi$
Calculate R
Is $R > R_l$
No
Yes
Is $R < R_l$
No
Yes
Is $X < X_1$
No
Is $X < X_2$
No
Delay
Delay
Send Trip Signal
Jump

Generalised Interface for Distance Relays

- The schematic diagram of the generalised inter face for realisation of different functions, like data acquisition and processing under an elaborate program control. At the core of this program is a signal processing algorithm which processes the incoming digitised relaying data to detect the fault. Recently, strong interest in applying digital techniques to protective relaying has been indicated by many publications in this field. Most of the work reported in journals concentrates on the development of digital signal processing in algorithms for relaying of EHV lines in general distance relaying of transmission lines have been derived during the past two decades and a comparative evaluation of different algorithms for line relaying has also been reported.

- The aim of the most of these algorithm is to extract the fundamental frequency components from the complex post-fault voltage and current signals containing a transient dc offset component and harmonic frequency components in addition to the power frequency fundamental component. As the microprocessor requires computationally simply and fast algorithm in order to perform the relaying function, only a few algorithms are suitable for microprocessor implementation. Algorithms which are suitable for microprocessor implementation are described in subsequent sections. These algorithm are based on the solution of differential equation, discrete Fourier trancsform, Walsh-hadamard transform and rationalised Haar transform techniques.

- The exponentially decaying dc offset present in the relaying signals gives rise to fairly large errors in the phasor estimates unless the ofsset terms are removed prior to the execution of the algorithms. Data window is the time span covered by the sample set needed to execute the algorithm. Fourier, Walsh and Haar algorithms can either be a full-cycle window or a half-cycle window. The full-cycle data window increases the operating time of the relays to more than cycle. Since the distance relay's operation within a cycle after the fault inception is desirable, the half cycle data window is preferable. The half-cycle window Fourier, Walsh and Haar algorithms in prticular particular require that the dc offset be removed prior to processing, while the differential equation algorithms ideallydo not require its elimination. Therfore, the removal of dc offset component from the relaying signals is also discussed in sectionnext.

Differential Equation Technique

- The differential equation algorithm is based on a model of the system rather thanon a model of the signal. This algorithm for the computation of line parameters, i.e. R and X is based on the solution of the differentioal equation representing the transmission line model. The transmission line is modelled as a series R-L circuit resulting in the following differential equation.

$$V = Ri + L\frac{di}{dt}$$

- This relationship holds for both steady-state and transient cinditions. The solution for R and L is accomplished by numerical differentiation over two successive time periods. Then the solution of the resulting simultaneous linear equations are abotined.

The derivation of the numerical algorithm for the circuit parametrs is a straight forward application of differences. Simultaneous samples of both voltage and current sigals are taken at a fixed rate, at times t_0 , and t_2 thus, there are two sample periods, the first from t_0 to t_1 is labelled A; the second from t_1 to t_2 is labelled B. if V_0, V_1, V_2 are the three consecutive samples of voltage signal and i_0 , i_1 , i_2 the three consecutive samples of current signal, the average values during the periods A and B are as follows.

$$i_A = \frac{i_0 + i_1}{2} \qquad i_B = \frac{i_1 + i_2}{2}$$

$$V_A = \frac{V_0 + V_1}{2} \qquad V_B = \frac{V_1 + V_2}{2}$$

$$\frac{di_A}{dt} = \frac{i_1 - i_0}{t_1 - t_0} \qquad \frac{di_B}{dt} = \frac{i_2 - i_1}{t_2 - t_1}$$

- Now for these two periods, the following diferential equations hold.

$$V_A = Ri_A + L\frac{di_A}{dt}$$

$$V_B = Ri_B + L\frac{di_B}{dt}$$

- By substitution of the know values, there remain two algebraic equations with two unknowns ,R and L. the solution of the simultaneous Eqs (8.17) and (8.18) gives the expressions for R and L. for the calculatin algorithm, the following difinitions are made.

$$CA = i_0 + i_1 \qquad CB = i_1 + i_2$$

$$VA = V_0 + V_1 \qquad VB = V_1 + V_2$$

$$DA = i_1 - i_0 \qquad DB = i_2 - i_1$$

$$DT = t_2 - t_1 = t_1 - t_0$$

Then
$$R = \frac{DA * VB - DB * VA}{DA * CB - DB * CA}$$

$$L = \frac{DT}{2}\frac{VA * CB - VB * CA}{DA * CB - DB * CA}$$

- The line reactance $X(=wL)$ is proportional to the line inductance L .

Then
$$X = w\frac{DT}{2}\frac{VA*CB-VB*CA}{DA*CB-DB*AC}$$

- The Eq(8.19) and (8.21) are programmed to determine R and X from the sampled values of voltage and current.
- As this algorithm requires a very short data window of three-samples only, it is very fast and is most suitable for microprocessor implementation. The program flowchart for the computation of the R and X using this algorithm is shown in Fig. 8.27.

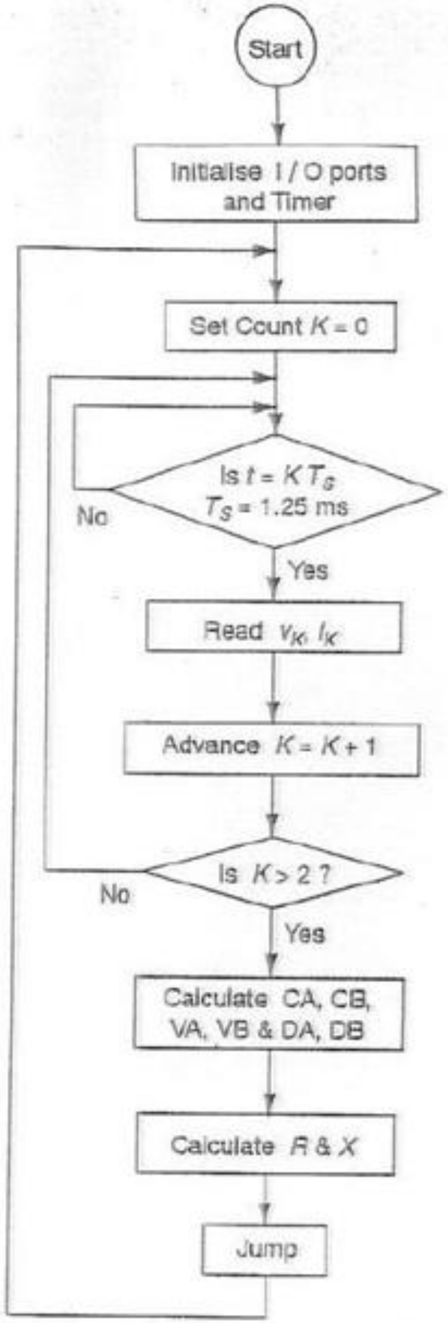


FIGURE 6.27 Program flowchart for computation of R and X

Discrete Fourier Transform(DFT)

- DFT is a algorithm for extracting the fundamental frequency components of fault signal
- DFT is used to evaluate the Fourier coefficients
- Using the DFT, the real and imaginary components of fundamental frequency voltage and current phasors are calculated
- then The real and imaginary components of apparent impedance (R, X) is calculated from these four quantities

- N sample of$X(t)$ taken at times $t = 0, T_S, \dots, (N-1)T_s$

where $T_s = {}^{T}/_{N}$ is the sampling interval and T is a period of $X(t)$

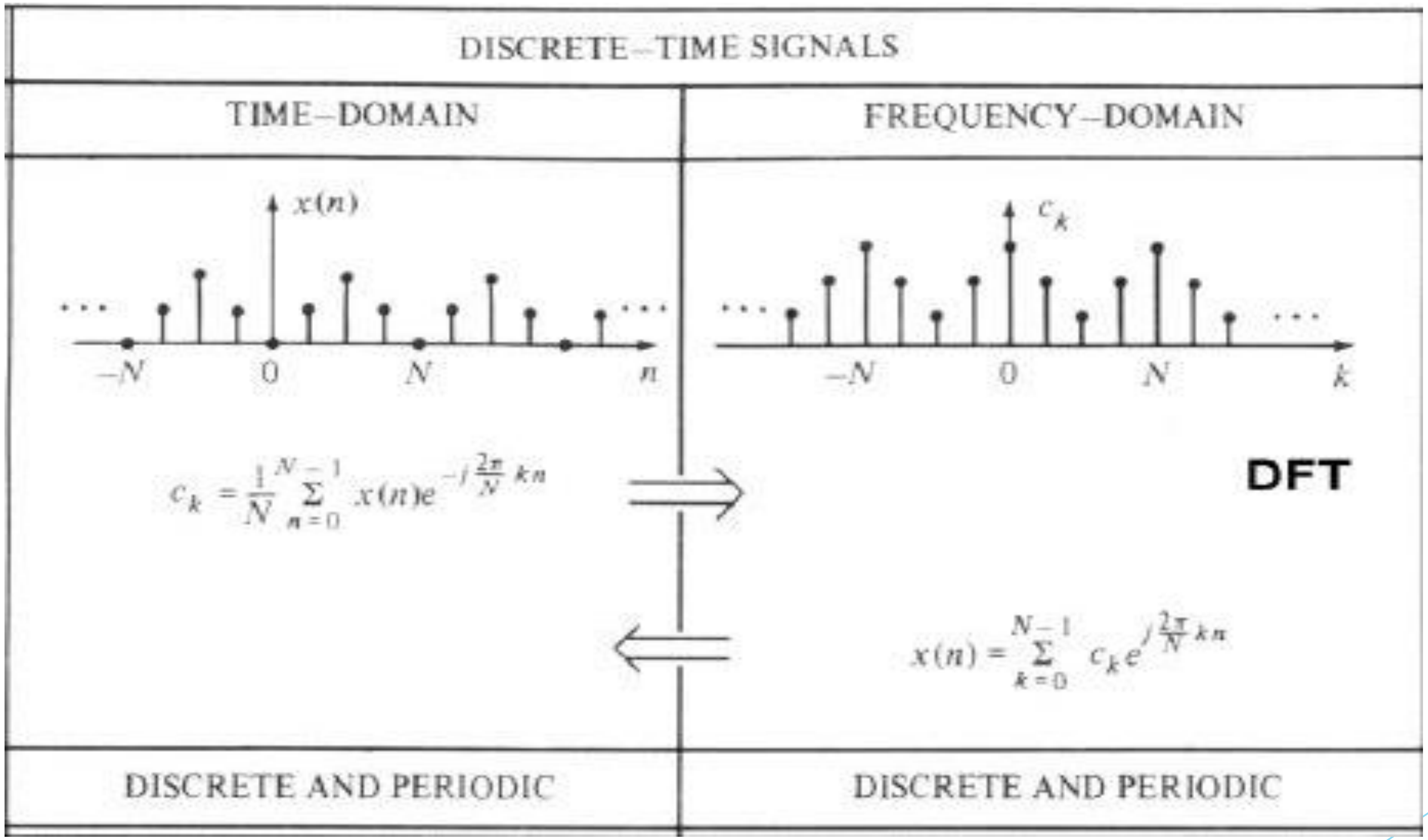
$$C_{fk} = \frac{1}{N}\sum_{m=0}^{N-1} x_m e^{-j2\pi km/N} \quad , \qquad K = 0,1,2,\dots,(N-1)$$

$$C_{fk} = \frac{1}{N}\sum_{m=0}^{N-1} x_m (\cos\frac{2\pi km}{N} - j\sin\frac{2\pi km}{N})$$

$$\frac{1}{2}(a_k - jb_k) = \frac{1}{N}\sum_{m=0}^{N-1} x_m (\cos\frac{2\pi km}{N} - j\sin\frac{2\pi km}{N}),$$

$$a_k = \frac{2}{N}\sum_{m=0}^{N-1} x_m (\cos\frac{2\pi km}{N}, \qquad K = 0,1,2,\dots,(N-1)$$

$$b_k = \frac{2}{N}\sum_{m=0}^{N-1} x_m \sin\frac{2\pi km}{N}, \qquad K = 0,1,2,\dots,(N-1)$$



DISCRETE–TIME SIGNALS
TIME–DOMAIN
FREQUENCY–DOMAIN
$x(n)$
$-N$ 0 N n
c_k
$-N$ 0 N k
$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$
DFT
$x(n) = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}$
DISCRETE AND PERIODIC
DISCRETE AND PERIODIC

DFT matrix representation

- Inputs: $$[X]=\left[x_0,x_1,x_2,\ldots,X_{(N-1)}\right]^T$$
- Outputs: $$\left[C_f\right]=\left[Cf \mid C_{f0},C_{f1},C_{f2},\ldots,C_{f(N-1)}\right]^T$$
- If $W=e^{-j2\pi/N}$

$$C_{fk}=\frac{1}{N}\sum_{m=0}^{N-1}x_m e^{-j2\pi km/N} \quad C_{fk}=\frac{1}{N}\sum_{m=0}^{\infty}x_m W^{km} \quad ,K=0,1,2,\ldots,(N-1)$$

- If W^E is the DFT matrix with row numbers k=0,1,2,…,N-1

and column numbers m=0,1,2,…,N-1 and $W^{E(k,m)}$ is in row k and column m:

$$\left[C_f\right]=\frac{1}{N}\left[W^E\right][X]$$

Fundamental frequency components of voltage and current

- Reduces the number of arithmetic operations and memory required to compute the DFT.

  Based on manipulation and factorization of DFT matrix W^E

  When N(sampling number) is 2^n (called power of 2 FFT) or

  N having n integral factors(called mixed radix transform) are the easiest cases for this algorithm.

  The main advantage is that reduces the computation and the main disadvantage is that complex arithmetic is involved

  FFT is useful in digital signal processing but not in digital relaying because in relaying application, N is small(4 t0 20 for most algorithms) and only a few C_{fk} are wanted.

Fundamental frequency components of voltage and current

- The real and imaginary components of the fundamental frequency (k=1) is:

$$b_1 = \sqrt{2}F_1 = \frac{2}{N}\sum_{m=0}^{N-1} x_m \sin\frac{2\pi m}{N}$$

$$a_1 = \sqrt{2}F_2 = \frac{2}{N}\sum_{m=0}^{N-1} x_m \cos\frac{2\pi m}{N}$$

- The real and imaginary components of the fundamental frequency voltage and current phasors by using the sampled values of them are:

$$F_{1(v)} = V_s, \qquad F_{2(v)} = V_c$$

$$F_{1(i)} = I_s, \qquad F_{2(i)} = I_c$$

Full-cycle data window DFT algorithm

- In this case both voltage and current signals are sampled simultaneously to aquire N samles for each signal if Vm and Im be the sampled values:

$$F_{1(v)} = V_s = \frac{\sqrt{2}}{N} \sum_{m=0}^{N-1} v_m \sin \frac{2\pi m}{N}$$

$$F_{2(v)} = V_c = \frac{\sqrt{2}}{N} \sum_{m=0}^{N-1} v_m \cos \frac{2\pi m}{N}$$

$$F_{1(i)} = I_s = \frac{\sqrt{2}}{N} \sum_{m=0}^{N-1} i_m \sin \frac{2\pi m}{N}$$

$$F_{2(i)} = I_c = \frac{\sqrt{2}}{N} \sum_{m=0}^{N-1} i_m \cos \frac{2\pi m}{N}$$

Half-cycle data window DFT algorithm

- In this case, the transient dc off set component is filtered out from the incoming row data samples prior to they're being processed
- After filtering the dc offset, the fault voltage and current waves contain only odd harmonics and half-wave symmetry
- the settling time is around half a cycle but display poor accuracy

$$V_c = \frac{2\sqrt{2}}{N} \sum_{m=0}^{\frac{N}{2}-1} v_m \cos \frac{2\pi m}{N}$$

$$V_s = \frac{2\sqrt{2}}{N} \sum_{m=0}^{\frac{N}{2}-1} v_m \sin \frac{2\pi m}{N}$$

$$I_s = \frac{2\sqrt{2}}{N} \sum_{m=0}^{\frac{N}{2}-1} i_m \sin \frac{2\pi m}{N}$$

$$I_c = \frac{2\sqrt{2}}{N} \sum_{m=0}^{\frac{N}{2}-1} i_m \cos \frac{2\pi m}{N}$$

Computation of apparent impedance

- The main objective of digital distance relaying is to calculate the apparent impedance of the line from relay location to the fault point in order to determine whether the fault lies within the relay protective zone or not
- For distance relaying the half –cycle data window is preferable.
- Phasor representation of fundamental frequency components of voltage and current signals are expressed in complex form:

$$V = V_s + jV_c \qquad I = I_s + jI_c$$

- Then the magnitudes(rms values) and phase angles of them are

$$V = \sqrt{V_s^2 + V_c^2} \qquad I = \sqrt{I_s^2 + I_c^2}$$

$$\Phi_v = \tan^{-1}\frac{V_c}{V_s} \qquad \Phi_i = \tan^{-1}\frac{I_c}{I_s}$$

- The apparent impedance is then given by

$$Z = \frac{V}{I} = \frac{|V|\angle\Phi_v}{|I|\angle\Phi_i} = \frac{\sqrt{V_s^2 + V_c^2}}{\sqrt{I_s^2 + I_c^2}}\angle(\Phi_v - \Phi_i)$$

- To simplify the calculation and reduce the time of it , Z is calculated as follows

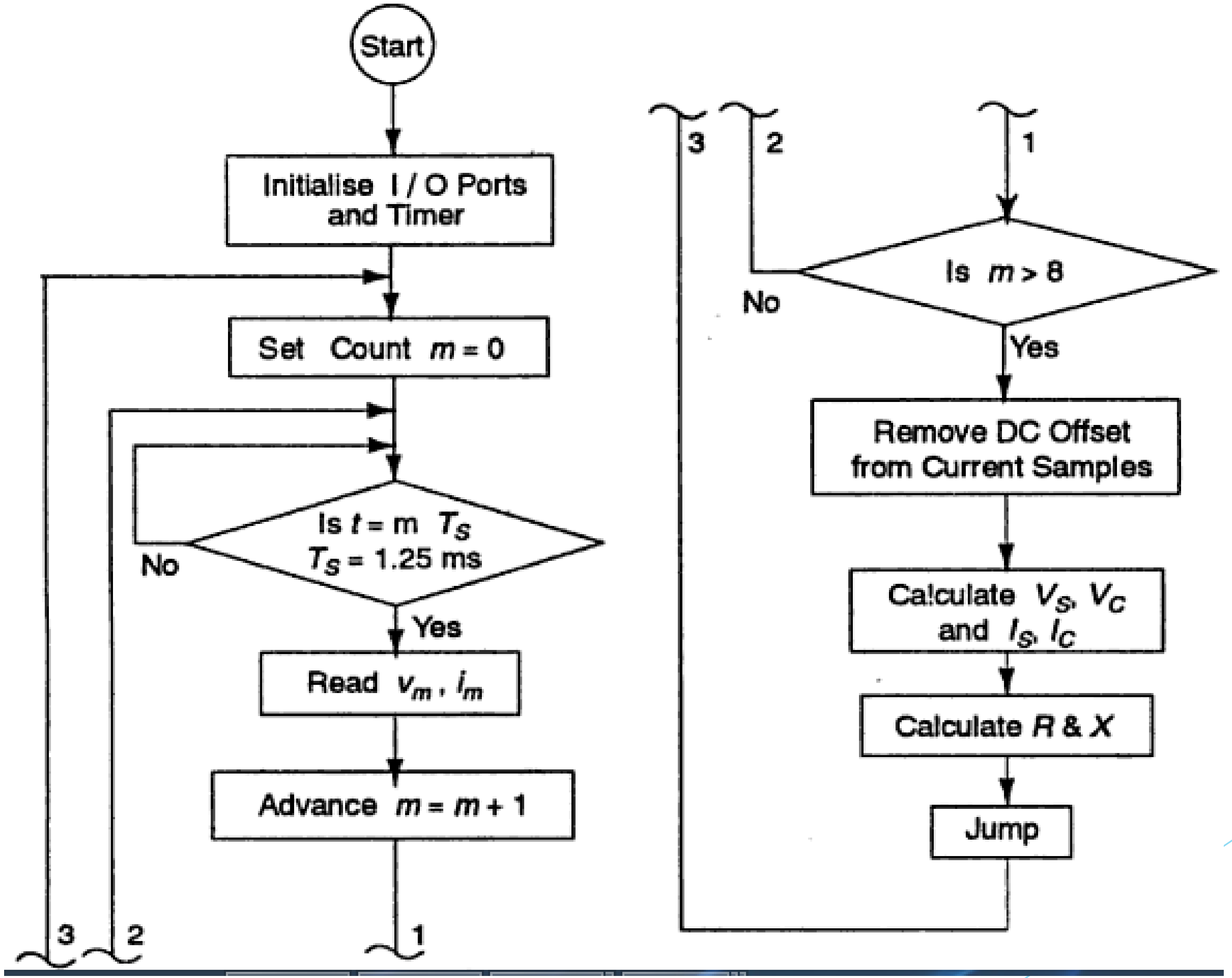
$$Z = \frac{V_s + jV_c}{I_s + jI_s} = \frac{V_s I_s + V_c I_c}{I_s^2 + I_c^2} + j\frac{V_c I_s - V_s I_c}{I_s^2 + I_c^2}$$

$$= R + jX$$

$$R = \frac{V_s I_s + V_c I_c}{I_s^2 + I_c^2}$$

$$X = \frac{V_c I_s - V_s I_c}{I_s^2 + I_c^2}$$

- The program flowchart for computation of R and X using half-cycle data window DFT algorithm is shown as follows:



Walsh function

- Walsh functions are a complete set of functions which form an ordered set of rectangular wave forms taking only two amplitude values namely -1 , 1.
- The function is wal(k,t) that t is time period and k is the number of zero crossings in t.
- The frequency of such such functions is known as sequency.

  Sequency can be expressed in terms of zero crossings(n_{zc}) per unit interval.

- Walsh functions can be rearranged in ordering schemes such as walsh or sequency order and hadamard or natural order. they can transform to each other.

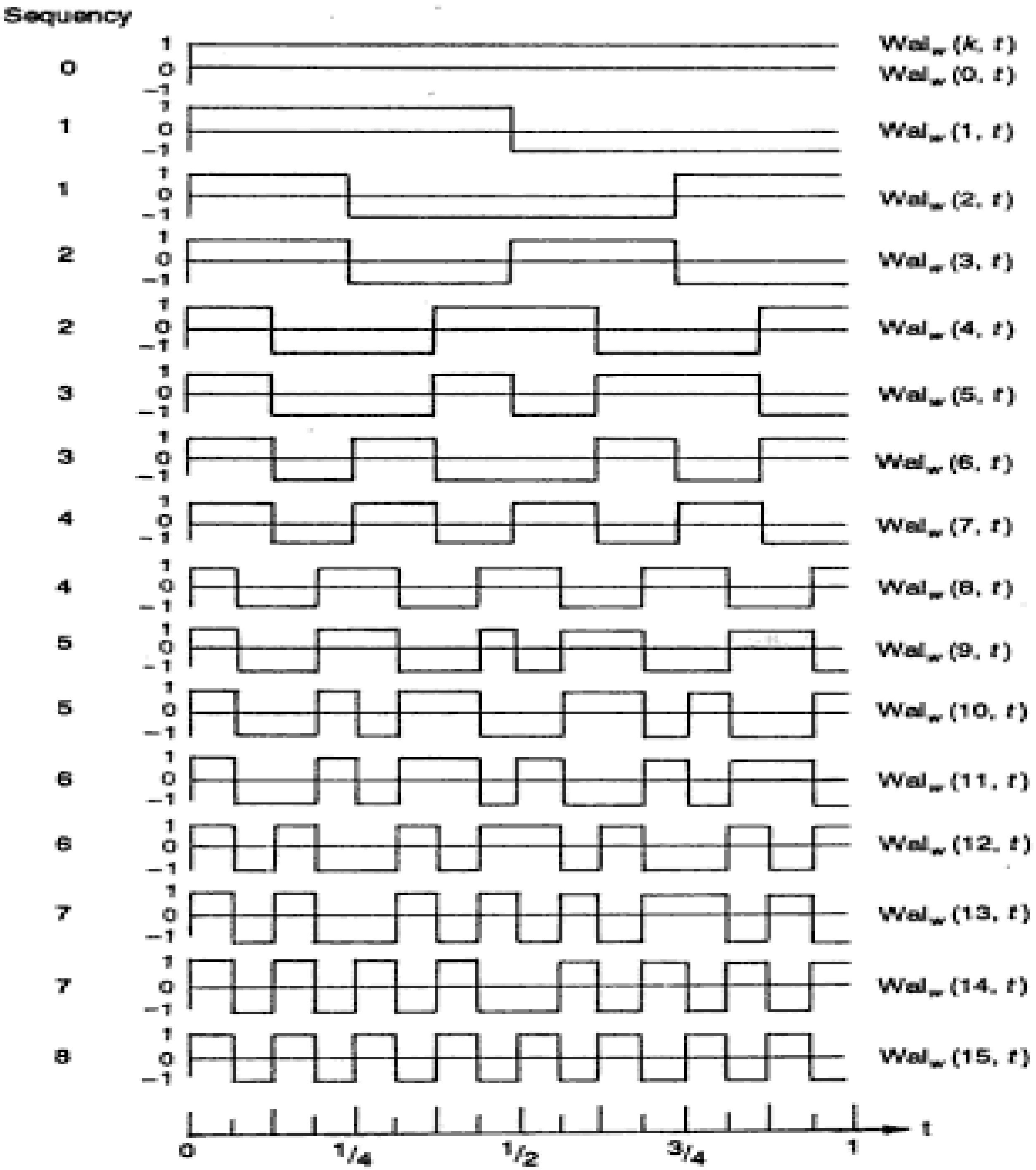
$$\text{Sequency} = \begin{cases} 0 & \text{for } n_{zc} = 0 \\ n_{zc}/2 & \text{for even } n_{zc} \\ (n_{zc}+1)/2 & \text{for odd } n_{zc} \end{cases}$$

Walsh or sequency ordering

- This set of walsh function is denoted by

$$S_w = \{Wal_w(k,t), k = 0,1,2,\ldots,(N-1)\}$$

- Where $N = 2^n$, $n = 0,1,2,\ldots,w$ denotes walsh ordering, k denotes the K_{th} member of Sw or number of zero crossings.


Sequency
0
1
1
2
2
3
3
4
4
5
5
6
6
7
7
8
$\mathrm{Wal}_w(k, t)$
$\mathrm{Wal}_w(0, t)$
$\mathrm{Wal}_w(1, t)$
$\mathrm{Wal}_w(2, t)$
$\mathrm{Wal}_w(3, t)$
$\mathrm{Wal}_w(4, t)$
$\mathrm{Wal}_w(5, t)$
$\mathrm{Wal}_w(6, t)$
$\mathrm{Wal}_w(7, t)$
$\mathrm{Wal}_w(8, t)$
$\mathrm{Wal}_w(9, t)$
$\mathrm{Wal}_w(10, t)$
$\mathrm{Wal}_w(11, t)$
$\mathrm{Wal}_w(12, t)$
$\mathrm{Wal}_w(13, t)$
$\mathrm{Wal}_w(14, t)$
$\mathrm{Wal}_w(15, t)$
0
1/4
1/2
3/4
1
t

Hadamard or natural ordering

- This set of walsh function is denoted by

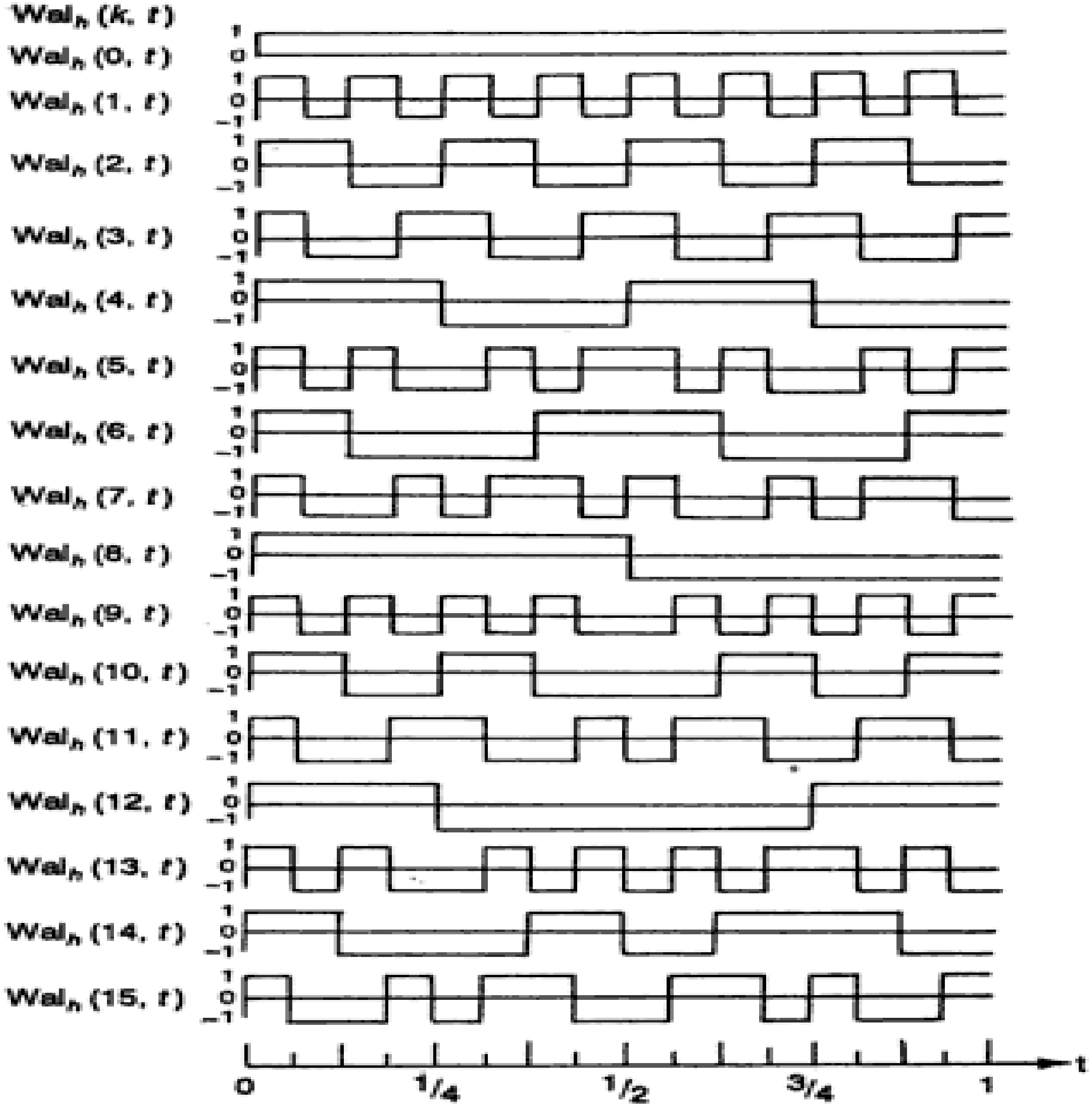
  $$S_h = \{Wal_h(k,t), k = 0,1,2,.....,(N-1)\}$$

  Where $N = 2^n, n = 0,1,2,...,h$ denotes hadamard ordering, K denotes the K_{th} member of S_h or number of zero crossings.

  The relation between hadamard ordered functions and walsh ordered functions is :

  $$Wal_h(k,t) = Wal_w\left[b(\le k \ge),t\right]$$

  Where $\le k \ge$ is obtained by bit reversal of k and $b(\le k \ge)$ is the gray code to binary conversion of $\le k \ge$



$Wal_h(k, t)$
$Wal_h(0, t)$
$Wal_h(1, t)$
$Wal_h(2, t)$
$Wal_h(3, t)$
$Wal_h(4, t)$
$Wal_h(5, t)$
$Wal_h(6, t)$
$Wal_h(7, t)$
$Wal_h(8, t)$
$Wal_h(9, t)$
$Wal_h(10, t)$
$Wal_h(11, t)$
$Wal_h(12, t)$
$Wal_h(13, t)$
$Wal_h(14, t)$
$Wal_h(15, t)$
1
0
−1
0
1/4
1/2
3/4
1
t

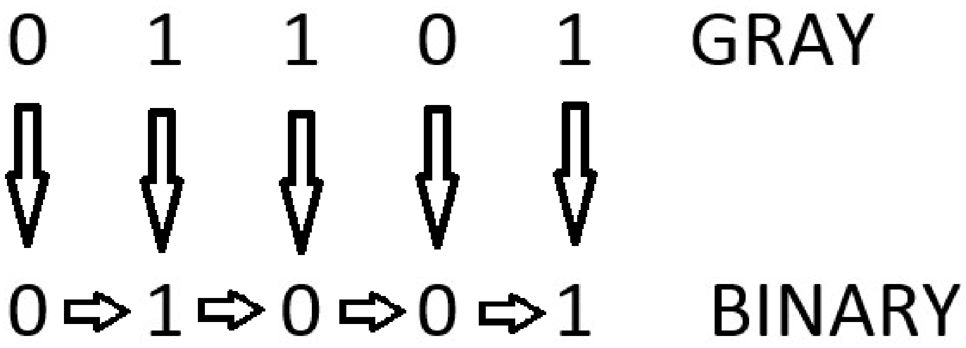
$K_{decimal}$	K_{binary}	$<K>_{binary}$	$b(<K>)_{binary}$	$b(<K>)_{decimal}$	Equation 8.77
0	0000	0000	0000 8421	0	$\mathrm{Wal}_h(0, t) = \mathrm{Wal}_w(0, t)$
1	0001	1000	1111	15	$\mathrm{Wal}_h(1, t) = \mathrm{Wal}_w(15, t)$
2	0010	0100	0111	7	$\mathrm{Wal}_h(2, t) = \mathrm{Wal}_w(7, t)$
3	0011	1100	1000	8	$\mathrm{Wal}_h(3, t) = \mathrm{Wal}_w(8, t)$
4	0100	0010	0011	3	$\mathrm{Wal}_h(4, t) = \mathrm{Wal}_w(3, t)$
5	0101	1010	1100	12	$\mathrm{Wal}_h(5, t) = \mathrm{Wal}_w(12, t)$
6	0110	0110	0100	4	$\mathrm{Wal}_h(6, t) = \mathrm{Wal}_w(4, t)$
7	0111	1110	1011	11	$\mathrm{Wal}_h(7, t) = \mathrm{Wal}_w(11, t)$
8	1000	0001	0001	1	$\mathrm{Wal}_h(8, t) = \mathrm{Wal}_w(1, t)$
9	1001	1001	1110	14	$\mathrm{Wal}_h(9, t) = \mathrm{Wal}_w(14, t)$
10	1010	0101	0110	6	$\mathrm{Wal}_h(10, t) = \mathrm{Wal}_w(6, t)$
11	1011	1101	1001	9	$\mathrm{Wal}_h(11, t) = \mathrm{Wal}_w(9, t)$
12	1100	0011	0010	2	$\mathrm{Wal}_h(12, t) = \mathrm{Wal}_w(2, t)$
13	1101	1011	1101	13	$\mathrm{Wal}_h(13, t) = \mathrm{Wal}_w(13, t)$
14	1110	0111	0101	5	$\mathrm{Wal}_h(14, t) = \mathrm{Wal}_w(5, t)$
15	1111	1111	1010	10	$\mathrm{Wal}_h(15, t) = \mathrm{Wal}_w(10, t)$

Gray code to binary conversion

- The M.S.B of the binery number will be the M.S.B of the given grey code.
- Now if the second grey bit is 0 the second bunery bit will be as the previous or the first bit. If the grey bit is 1 the second binery bit will alter.if it was 1 it will be 0 and if it was 0 it will be 1.
- This step is continued for all the bits to do grey code to binery conversion.

One example given below will make your idea clear.

let the grey code be 01101



Walsh-hadamard matrices

- In the discrete case, uniform sampling of the set of N walsh function of any ordering at N equidistant points results in the (N*N) walsh-hadamard matrices of corresponding order
- the (N*N) walsh-hadamard matrices in walsh and hadamard orders are denoted by $[H_w(n)]$ and $[H_h(n)]$ respectively, where

$$n = \log_2^N$$

- Each row of the matrix is a discrete walsh function.

$$
[H_w(4)] = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{bmatrix}
$$

$$[H_h(4)] = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1
\end{bmatrix}$$

Fourier-walsh theory

The fourier expantion:

$$x(t) = F_0 + \sqrt{2}F_1 \sin\frac{2\pi t}{T} + \sqrt{2}F_2 \cos\frac{2\pi t}{T} + \sqrt{2}F_3 \sin\frac{4\pi t}{T} + \sqrt{2}F_4 \cos\frac{4\pi t}{T} + \ldots$$

$$F_0 = \frac{1}{T}\int_0^T x(t)dt$$

$$F_1 = \frac{\sqrt{2}}{T}\int_0^T x(t)\sin\frac{2\pi t}{T}dt$$

$$F_2 = \frac{\sqrt{2}}{T}\int_0^T x(t)\cos\frac{2\pi t}{T}dt$$

The Walsh expansion of the signal $x(t)$ is defined as

$$x(t) = \sum_{k=0}^{\infty} W_{wk} Wal_w(k, \frac{1}{T})$$

Where, the Walsh coefficients W_{wk} are given by

$$W_{wk} = \frac{1}{T}\int_0^T x(t) Wal_w(k, \frac{t}{T})$$

The Fourier and Walsh coefficients vectors are given by

$$[F]=[F_0,F_1,F_2,\ldots,F_{N-1}]^T$$

$$[W]=\left[W_{w0},W_{w1},W_{w2},\ldots,W_{w(N-1)}\right]^T$$

By substituting the fourier expantion in the Walsh coefficients equation:

$$[W_w]=[A][F]$$

Since $[A]$ is an orthogonal matrix, $[A]^{-1}=[A]^T$:

$$[F]=[A]^T[W_w]$$

The fundamental fourier coefficients F_1,F_2 are as follows:

$$F_1=0.9W_{w1}-0.3W_{w5}-0.074W_{w9}-0.179W_{w13}$$

$$F_2=0.9W_{w2}-0.373W_{w6}-0.074W_{w10}-0.179W_{w14}$$

$$
[A] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9 & 0 & 0 & 0 & 0.30 & 0 & 0 & 0 & 0.180 & 0 & 0 & 0 & 0.129 & 0 & 0 \\
0 & 0 & 0.9 & 0 & 0 & 0 & -0.30 & 0 & 0 & 0 & 0.180 & 0 & 0 & 0 & -0.129 & 0 \\
0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.30 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.30 & 0 & 0 & 0 \\
0 & -0.373 & 0 & 0 & 0 & 0.724 & 0 & 0 & 0 & 0.435 & 0 & 0 & 0 & -0.053 & 0 & 0 \\
0 & 0 & 0.373 & 0 & 0 & 0 & 0.724 & 0 & 0 & 0 & -0.435 & 0 & 0 & 0 & -0.053 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.074 & 0 & 0 & 0 & -0.484 & 0 & 0 & 0 & 0.65 & 0 & 0 & 0 & 0.269 & 0 & 0 \\
0 & 0 & -0.074 & 0 & 0 & 0 & 0.484 & 0 & 0 & 0 & 0.65 & 0 & 0 & 0 & -0.269 & 0 \\
0 & 0 & 0 & -0.373 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.724 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.373 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.724 & 0 & 0 & 0 \\
0 & -0.179 & 0 & 0 & 0 & 0.2005 & 0 & 0 & 0 & -0.269 & 0 & 0 & 0 & 0.65 & 0 & 0 \\
0 & 0 & 0.179 & 0 & 0 & 0 & 0.2005 & 0 & 0 & 0 & 0.269 & 0 & 0 & 0 & 0.65 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9
\end{bmatrix}
$$

- Walsh-ordered walsh-hadamard transform $(WHT)_w$:

$$[W_w] = \frac{1}{N}[H_W(n)][X]$$

  - $[W_w]$ Is the Walsh coefficients
  - $[H_w(n)]$ Is the Walsh-ordered Walsh-hadmard matrix
  - $[X]$ Is the data sequence $X_m, m = 1,2,\ldots\ldots,(N-1)$

- Hadamard_ordered Walsh-hadamard transform $(WHT)_w$:

$$[W_h] = \frac{1}{N}[H_h(n)][X]$$

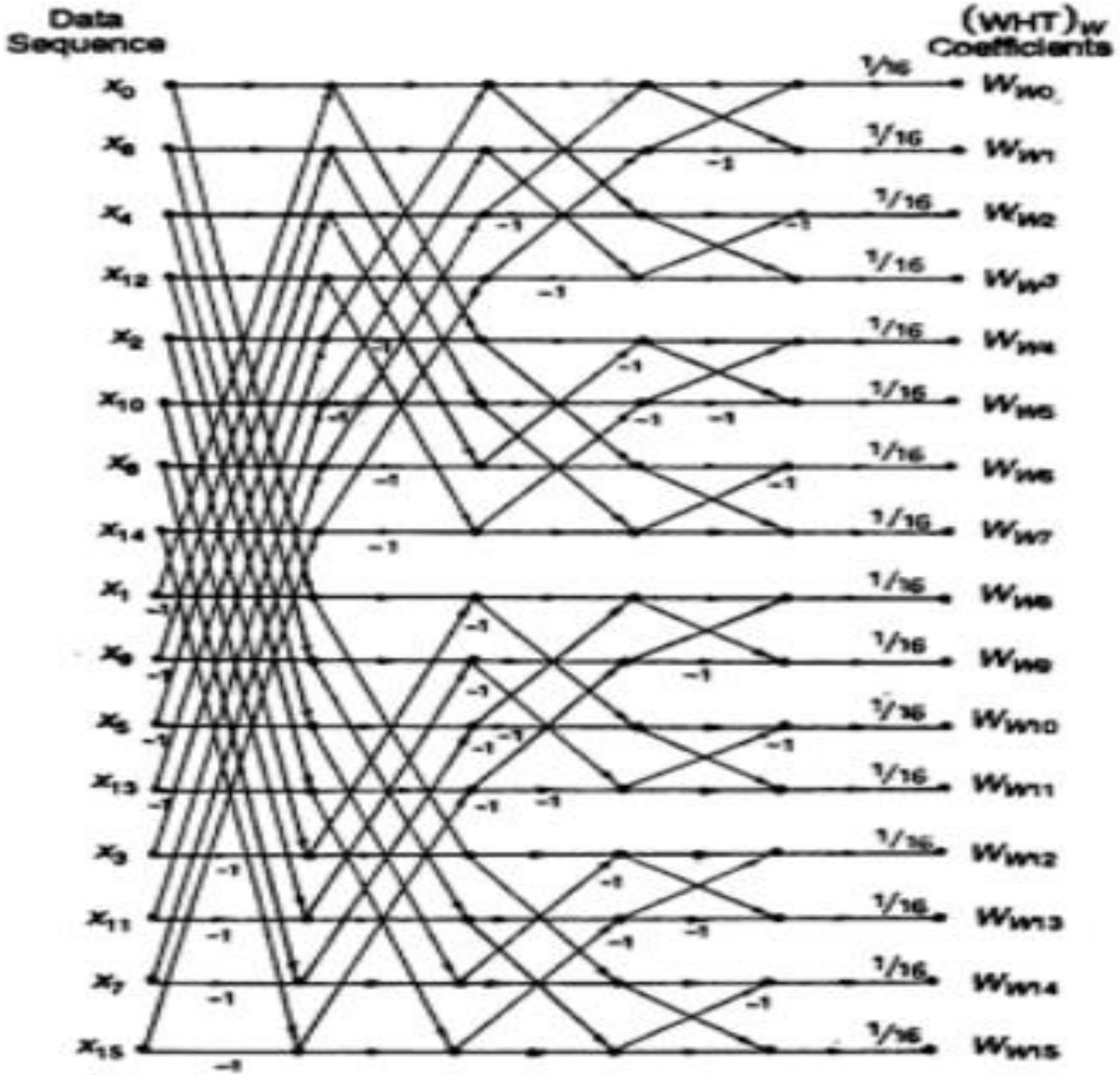
  - $[W_h]$ Is the hadamard cofficients
  - $[W_h(n)]$ Is the hadamard-ordered Walsh-hadamard matrix

Computation of walsh coefficients using Fast walsh-hadamard transform $(FWHT)$

- Based on matrix factoring or partitioning
- Direct computation of WHT coefficients using N operation whereas the FWHT reduces the computation to $N\log_2^N$ operation
- Factors characteristics:

1. The number of matrix factors and stages is $\log_2^N$
2. Any row of matrix factors contains only tow non-zero elements(+1/-1) which correspond to an addition/subtraction
3. Eash matrix factor corresponds to a stage in the reverse order
4. Total number of operation of all walsh coefficients is $N\log_2^N$

$$
\begin{aligned}
[H_w(4)] = &\left\{ diag\left[\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \right] \right\} \\
&\times \left\{ diag\left[\begin{pmatrix} U_2 & U_2 \\ U_2 & -U_2 \end{pmatrix}, \begin{pmatrix} U_2 & -U_2 \\ U_2 & U_2 \end{pmatrix}, \begin{pmatrix} U_2 & U_2 \\ U_2 & -U_2 \end{pmatrix}, \begin{pmatrix} U_2 & -U_2 \\ U_2 & U_2 \end{pmatrix} \right] \right\} \\
&\times \left\{ diag\left[\begin{pmatrix} U_4 & U_4 \\ U_4 & -U_4 \end{pmatrix}, \begin{pmatrix} U_4 & -U_4 \\ U_4 & U_4 \end{pmatrix} \right] \right\} \begin{pmatrix} U_8 & U_8 \\ U_8 & -U_8 \end{pmatrix} \left(U_{16}^{BRO} \right)
\end{aligned}
$$



Data Sequence
(WHT)w Coefficients
x0
x8
x4
x12
x2
x10
x6
x14
x1
x9
x5
x13
x3
x11
x7
x15
1/16
Ww0
Ww1
Ww2
Ww3
Ww4
Ww5
Ww6
Ww7
Ww8
Ww9
Ww10
Ww11
Ww12
Ww13
Ww14
Ww15
−1

Computation of walsh coefficients using Fast walsh-hadamard transform $(FWHT)$

1. using matix factoring:

$$[H_h(4)]$$

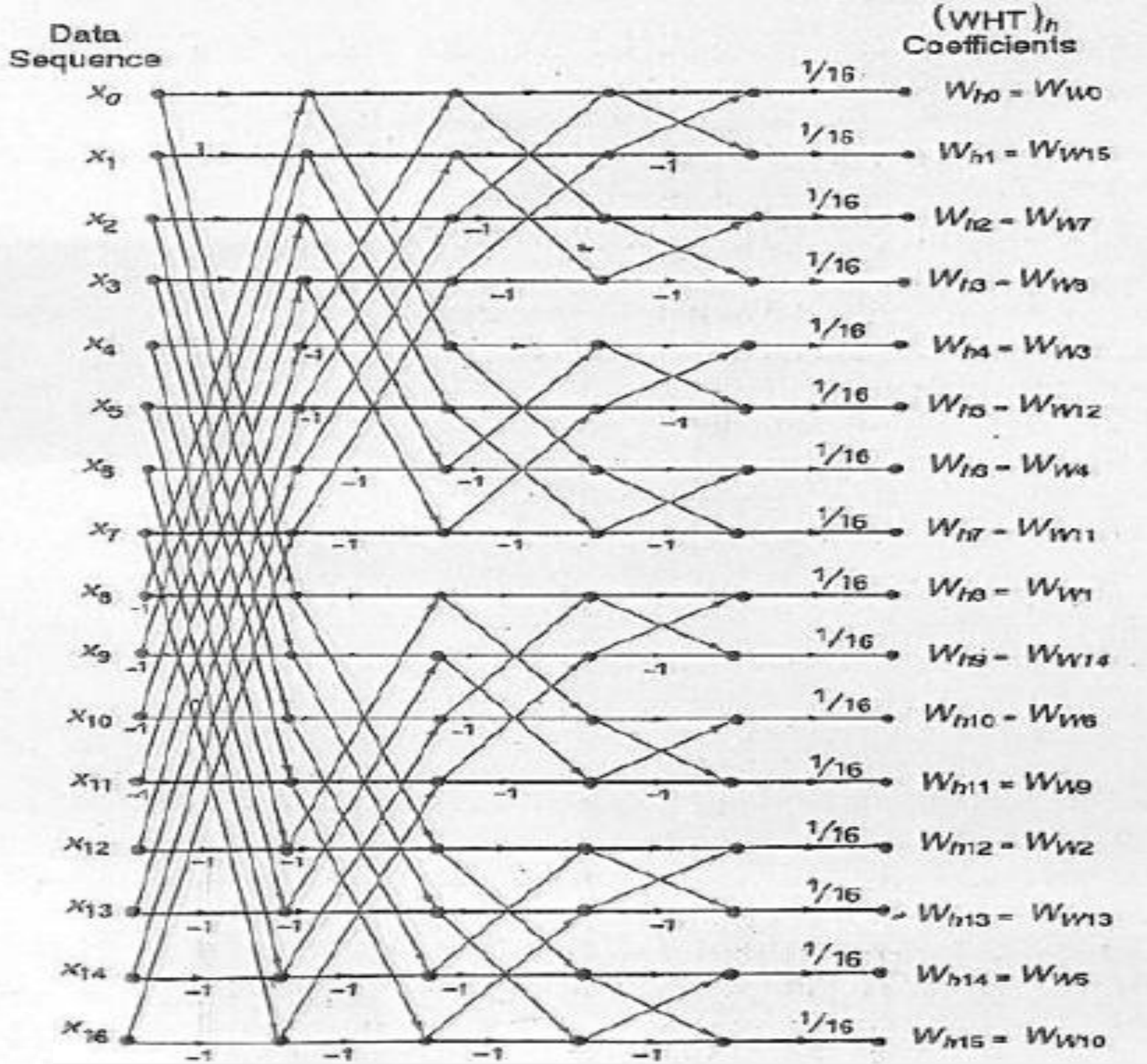
$$=(diag[[H_h(1)],[H_h(1)],[H_h(1)],[H_h(1)],[H_h(1)],[H_h(1)],[H_h(1)],[H_h(1)]]$$

$$\times(diag[[H_h(1)]\otimes U_2,[H_h(1)]\otimes U_2,[H_h(1)]\otimes U_2,[H_h(1)]\otimes U_2])$$

$$\times(diag[[H_h(1)]\otimes U_4,[H_h(1)]\otimes U_4])[[H_h(1)]\otimes U_8]$$

- Where $H_h(1) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
- Where $\otimes$ is the kronecker or direct product of matrices
- Examples

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 & 1\cdot 5 & 2\cdot 0 & 2\cdot 5 \\ 1\cdot 6 & 1\cdot 7 & 2\cdot 6 & 2\cdot 7 \\ 3\cdot 0 & 3\cdot 5 & 4\cdot 0 & 4\cdot 5 \\ 3\cdot 6 & 3\cdot 7 & 4\cdot 6 & 4\cdot 7 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 & 10 \\ 6 & 7 & 12 & 14 \\ 0 & 15 & 0 & 20 \\ 18 & 21 & 24 & 28 \end{bmatrix}$$



Data Sequence
(WHT)h Coefficients
x0
x1
x2
x3
x4
x5
x6
x7
x8
x9
x10
x11
x12
x13
x14
x16
1/16
Wh0 = Ww0
Wh1 = Ww15
Wh2 = Ww7
Wh3 = Ww8
Wh4 = Ww3
Wh5 = Ww12
Wh6 = Ww4
Wh7 = Ww11
Wh8 = Ww1
Wh9 = Ww14
Wh10 = Ww6
Wh11 = Ww9
Wh12 = Ww2
Wh13 = Ww13
Wh14 = Ww5
Wh15 = Ww10
−1

2. Using matrix partitioning:

$$B_x(3) = \frac{1}{8} H_h(3) X(3)$$

$$\begin{bmatrix} B_x(0) \\ B_x(1) \\ B_x(2) \\ B_x(3) \\ \hdashline B_x(4) \\ B_x(5) \\ B_x(6) \\ B_x(7) \end{bmatrix} = \frac{1}{8} \begin{bmatrix} \mathbf{H}_h(2) & \mathbf{H}_h(2) \\ \mathbf{H}_h(2) & -\mathbf{H}_h(2) \end{bmatrix} \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ \hdashline X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} B_x(0) \\ B_x(1) \\ B_x(2) \\ B_x(3) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(2) \begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix}$$

$$\begin{bmatrix} B_x(4) \\ B_x(5) \\ B_x(6) \\ B_x(7) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(2) \begin{bmatrix} X_1(4) \\ X_1(5) \\ X_1(6) \\ X_1(7) \end{bmatrix}$$

$$X_1(l) = X(l) + X(4 + l), \qquad l = 0, 1, 2, 3$$
$$X_1(l) = X(l - 4) - X(l), \qquad l = 4, 5, 6, 7$$

$$\begin{bmatrix} B_x(0) \\ B_x(1) \\ \hdashline B_x(2) \\ B_x(3) \end{bmatrix} = \frac{1}{8} \begin{bmatrix} \mathbf{H}_h(1) & \mathbf{H}_h(1) \\ \mathbf{H}_h(1) & -\mathbf{H}_h(1) \end{bmatrix} \begin{bmatrix} X_1(0) \\ X_1(1) \\ \hdashline X_1(2) \\ X_1(3) \end{bmatrix}$$

$$\begin{bmatrix} B_x(4) \\ B_x(5) \\ \hdashline B_x(6) \\ B_x(7) \end{bmatrix} = \frac{1}{8} \begin{bmatrix} \mathbf{H}_h(1) & \mathbf{H}_h(1) \\ \mathbf{H}_h(1) & -\mathbf{H}_h(1) \end{bmatrix} \begin{bmatrix} X_1(4) \\ X_1(5) \\ \hdashline X_1(6) \\ X_1(7) \end{bmatrix} \Rightarrow$$

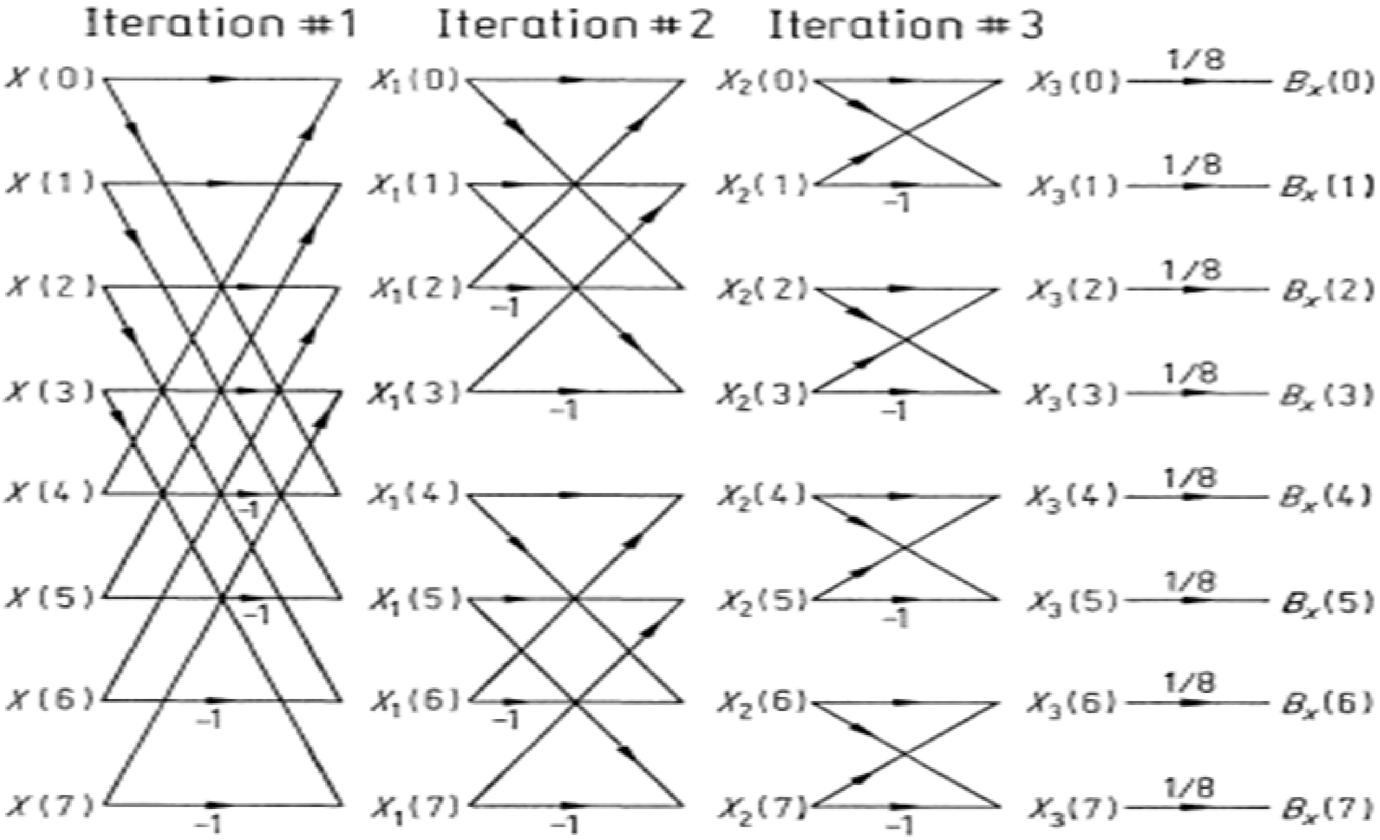
$$\begin{bmatrix} B_x(0) \\ B_x(1) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_1(0) + X_1(2) \\ X_1(1) + X_1(3) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_2(0) \\ X_2(1) \end{bmatrix}$$

$$\begin{bmatrix} B_x(2) \\ B_x(3) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_1(0) - X_1(2) \\ X_1(1) - X_1(3) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_2(2) \\ X_2(3) \end{bmatrix}$$

$$\begin{bmatrix} B_x(4) \\ B_x(5) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_1(4) + X_1(6) \\ X_1(5) + X_1(7) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_2(4) \\ X_2(5) \end{bmatrix}$$

$$\begin{bmatrix} B_x(6) \\ B_x(7) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_1(4) - X_1(6) \\ X_1(5) - X_1(7) \end{bmatrix} = \frac{1}{8} \mathbf{H}_h(1) \begin{bmatrix} X_2(6) \\ X_2(7) \end{bmatrix}$$

$$
\begin{aligned}
8B_x(0) &= X_2(0) + X_2(1) = X_3(0) \\
8B_x(1) &= X_2(0) - X_2(1) = X_3(1) \\
8B_x(2) &= X_2(2) + X_2(3) = X_3(2) \\
8B_x(3) &= X_2(2) - X_2(3) = X_3(3) \\
8B_x(4) &= X_2(4) + X_2(5) = X_3(4) \\
8B_x(5) &= X_2(4) - X_2(5) = X_3(5) \\
8B_x(6) &= X_2(6) + X_2(7) = X_3(6) \\
8B_x(7) &= X_2(6) - X_2(7) = X_3(7)
\end{aligned}
$$



- Example:

Let $\{X(m)\} = \{1\ 2\ 1\ 1\ 3\ 2\ 1\ 2\}$. Evaluate $W_x(k)$, $k = 0, 1, \ldots, 7$.

- Using : $(WHT)_v$

$$\begin{array}{ccc} & & \begin{array}{cccccccc} & & & & & (WHT)_v & & \end{array} \\ \begin{bmatrix} W_x(0) \\ W_x(1) \\ W_x(2) \\ W_x(3) \\ W_x(4) \\ W_x(5) \\ W_x(6) \\ W_x(7) \end{bmatrix} = \frac{1}{8} & & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} \end{array}$$

Evaluation of the above matrix equation leads to

$$\{W_x(k)\} = \left\{ \frac{13}{8}\quad \frac{-3}{8}\quad \frac{-1}{8}\quad \frac{3}{8}\quad \frac{1}{8}\quad \frac{-3}{8}\quad \frac{-1}{8}\quad \frac{-1}{8} \right\}$$

- USING $(FWHT)_h$:

Given the data sequence, $\{X(m)\} = \{1\ 2\ 1\ 1\ 3\ 2\ 1\ 2\}$. Use the $(\text{FWHT})_h$ to compute the $(\text{WHT})_h$ coefficients $B_x(k)$, $k = 0, 1, \ldots, 7$.

Solution:

$$
\begin{array}{llllll}
X(0) = 1 & X_1(0) = 4 & X_2(0) = 6 & X_3(0) = 13 & \xrightarrow{1/8} & B_x(0) \\
X(1) = 2 & X_1(1) = 4 & X_2(1) = 7 & X_3(1) = -1 & \xrightarrow{1/8} & B_x(1) \\
X(2) = 1 & X_1(2) = 2 & X_2(2) = 2 & X_3(2) = 3 & \xrightarrow{1/8} & B_x(2) \\
X(3) = 1 & X_1(3) = 3 & X_2(3) = 1 & X_3(3) = 1 & \xrightarrow{1/8} & B_x(3) \\
X(4) = 3 & X_1(4) = -2 & X_2(4) = -2 & X_3(4) = -3 & \xrightarrow{1/8} & B_x(4) \\
X(5) = 2 & X_1(5) = 0 & X_2(5) = -1 & X_3(5) = -1 & \xrightarrow{1/8} & B_x(5) \\
X(6) = 1 & X_1(6) = 0 & X_2(6) = -2 & X_3(6) = -1 & \xrightarrow{1/8} & B_x(6) \\
X(7) = 2 & X_1(7) = -1 & X_2(7) = 1 & X_3(7) = -3 & \xrightarrow{1/8} & B_x(7)
\end{array}
$$

Hence, $\{B_x(k)\} = \left\{\frac{13}{8}\ \ \frac{-1}{8}\ \ \frac{3}{8}\ \ \frac{1}{8}\ \ \frac{-3}{8}\ \ \frac{-1}{8}\ \ \frac{-1}{8}\ \ \frac{-3}{8}\right\}$

USING : $(FWHT)_W$

Example 6.5-1

Use the $(\text{FWHT})_w$ to find the $(\text{WHT})_w$ of $\{X(m)\} = \{1\ 2\ 1\ 1\ 3\ 2\ 1\ 2\}$.

Solution: If $\{\hat{X}(m)\} = \{\hat{X}(0)\ \hat{X}(1)\ \hat{X}(2)\ \hat{X}(3)\ \hat{X}(4)\ \hat{X}(5)\ \hat{X}(6)\ \hat{X}(7)\}$ denotes the sequence obtained by the bit-reversal of $\{X(m)\}$, then it follows that

$$\hat{X}(0) = X(0), \quad \hat{X}(1) = X(4), \quad \hat{X}(2) = X(2), \quad \hat{X}(3) = X(6),$$
$$\hat{X}(4) = X(1), \quad \hat{X}(5) = X(5), \quad \hat{X}(6) = X(3), \quad \hat{X}(7) = X(7)$$

Substituting for $\{X(m)\}$ and $\{\hat{X}(m)\}$ in Fig. 6.6, we obtain:

$$\begin{array}{lllll l}
X(0) = 1 & \hat{X}(0) = 1 & X_1(0) = 3 & X_2(0) = 5 & X_3(0) = 13 & \xrightarrow{1/8} W_x(0) \\
X(1) = 2 & \hat{X}(1) = 3 & X_1(1) = 5 & X_2(1) = 8 & X_3(1) = -3 & \xrightarrow{1/8} W_x(1) \\
X(2) = 1 & \hat{X}(2) = 1 & X_1(2) = 2 & X_2(2) = 1 & X_3(2) = -1 & \xrightarrow{1/8} W_x(2) \\
X(3) = 1 & \hat{X}(3) = 1 & X_1(3) = 3 & X_2(3) = 2 & X_3(3) = 3 & \xrightarrow{1/8} W_x(3) \\
X(4) = 3 & \hat{X}(4) = 2 & X_1(4) = -1 & X_2(4) = -1 & X_3(4) = 1 & \xrightarrow{1/8} W_x(4) \\
X(5) = 2 & \hat{X}(5) = 2 & X_1(5) = 1 & X_2(5) = 2 & X_3(5) = -3 & \xrightarrow{1/8} W_x(5) \\
X(6) = 1 & \hat{X}(6) = 1 & X_1(6) = 0 & X_2(6) = -1 & X_3(6) = -1 & \xrightarrow{1/8} W_x(6) \\
X(7) = 2 & \hat{X}(7) = 2 & X_1(7) = -1 & X_2(7) = 0 & X_3(7) = -1 & \xrightarrow{1/8} W_x(7)
\end{array}$$

Thus $$\{W_x(k)\} = \left\{\frac{13}{8}\ \frac{-3}{8}\ \frac{-1}{8}\ \frac{3}{8}\ \frac{1}{8}\ \frac{-3}{8}\ \frac{-1}{8}\ \frac{-1}{8}\right\}$$

Computation of apparent impedance

- Sampling both voltage and current signals over a full-cycle or half-cycle window (preferably half-cycle) $(FWHT)_H$
- Computation of walsh coefficients by using
- Computation of the fundamental fourier coefficients(F1 and F2) by

$$F_1 = 0.9W_{w1} - 0.373W_{w5} - 0.074W_{w9} - 0.179W_{w13}$$

$$F_2 = 0.9W_{w2} + 0.373W_{w6} - 0.074W_{w10} + 0.179W_{w14}$$

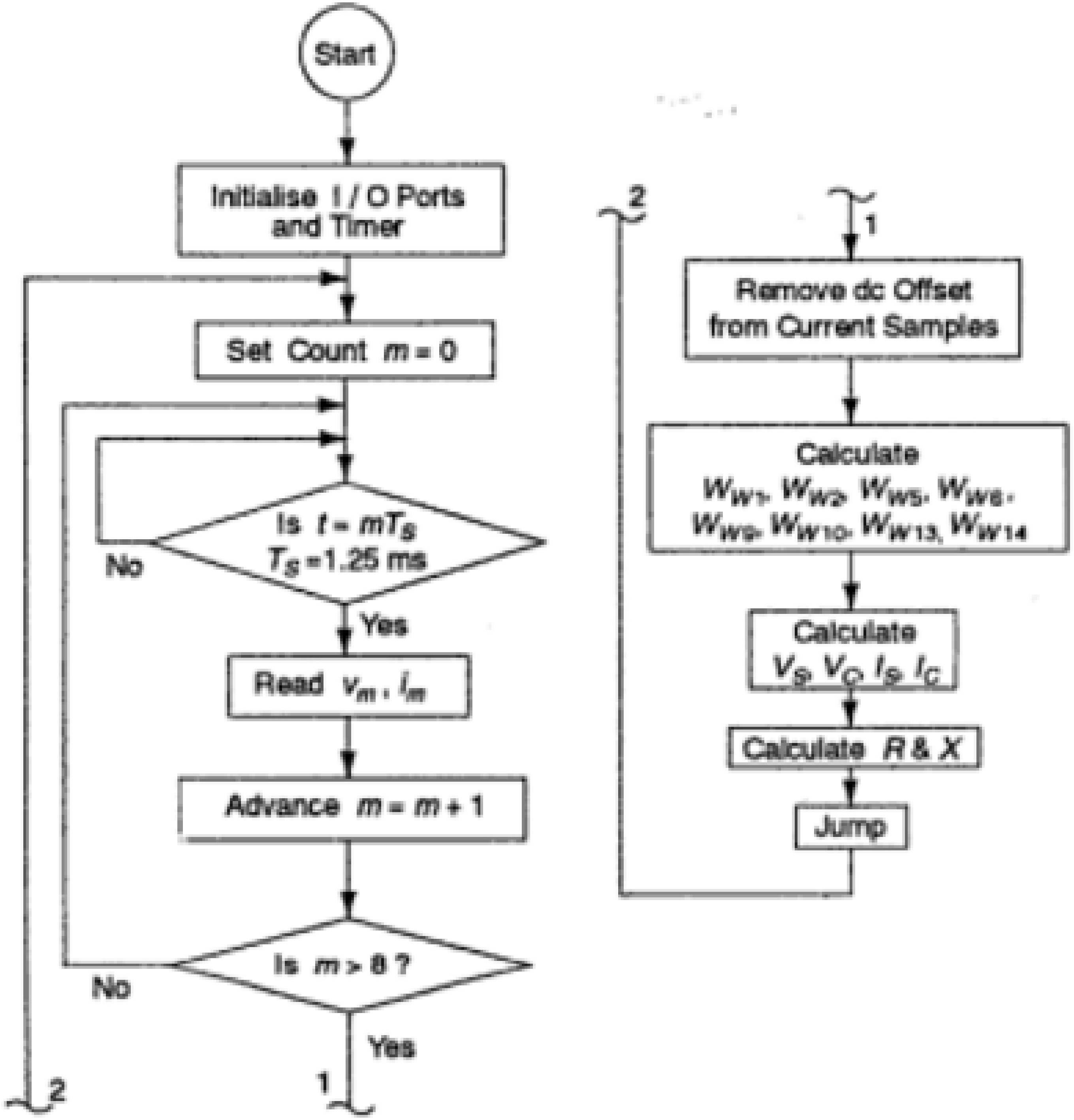
- then

$$F_{1(v)} = V_s, \qquad F_{2(v)} = V_c$$

$$F_{1(i)} = I_s, \qquad F_{2(i)} = I_c$$

$$\Rightarrow \quad V = V_s + jV_c \qquad I = I_s + jI_c$$

$$Z = \frac{V}{I} = \frac{|V|\angle\Phi_v}{|I|\angle\Phi_i} = \frac{\sqrt{V_s^2 + V_c^2}}{\sqrt{I_s^2 + I_c^2}}\angle(\Phi_v - \Phi_i)$$



Start
Initialise I / O Ports and Timer
Set Count m = 0
Is t = mT_S
T_S = 1.25 ms
No
Yes
Read v_m, i_m
Advance m = m + 1
Is m > 8 ?
No
Yes
1
2
Remove dc Offset from Current Samples
Calculate
W_W1, W_W2, W_W5, W_W6, W_W9, W_W10, W_W13, W_W14
Calculate
V_S, V_C, I_S, I_C
Calculate R & X
Jump

Haar transform(HT)

- Based on Haar functions which are periodic and orthogonal
- A recurrence relation to generate the set of N Haar functions har(r,m,t) is:

$$\text{har}(0,0,t) = 1 \qquad 0 \le t < 1$$

$$\text{har}(r,m,t) = \begin{cases} 2^{r/2} & \frac{m-1}{2^r} \le t < \frac{m-1/2}{2^r} \\ -2^{r/2} & \frac{m-1/2}{2^r} \le t < \frac{m}{2^r} \\ 0 & \text{elsewhere for } 0 \le t < 1 \end{cases}$$

Where $0 \le r \le \log_2^N$ and $1 \le m \le 2^r$.

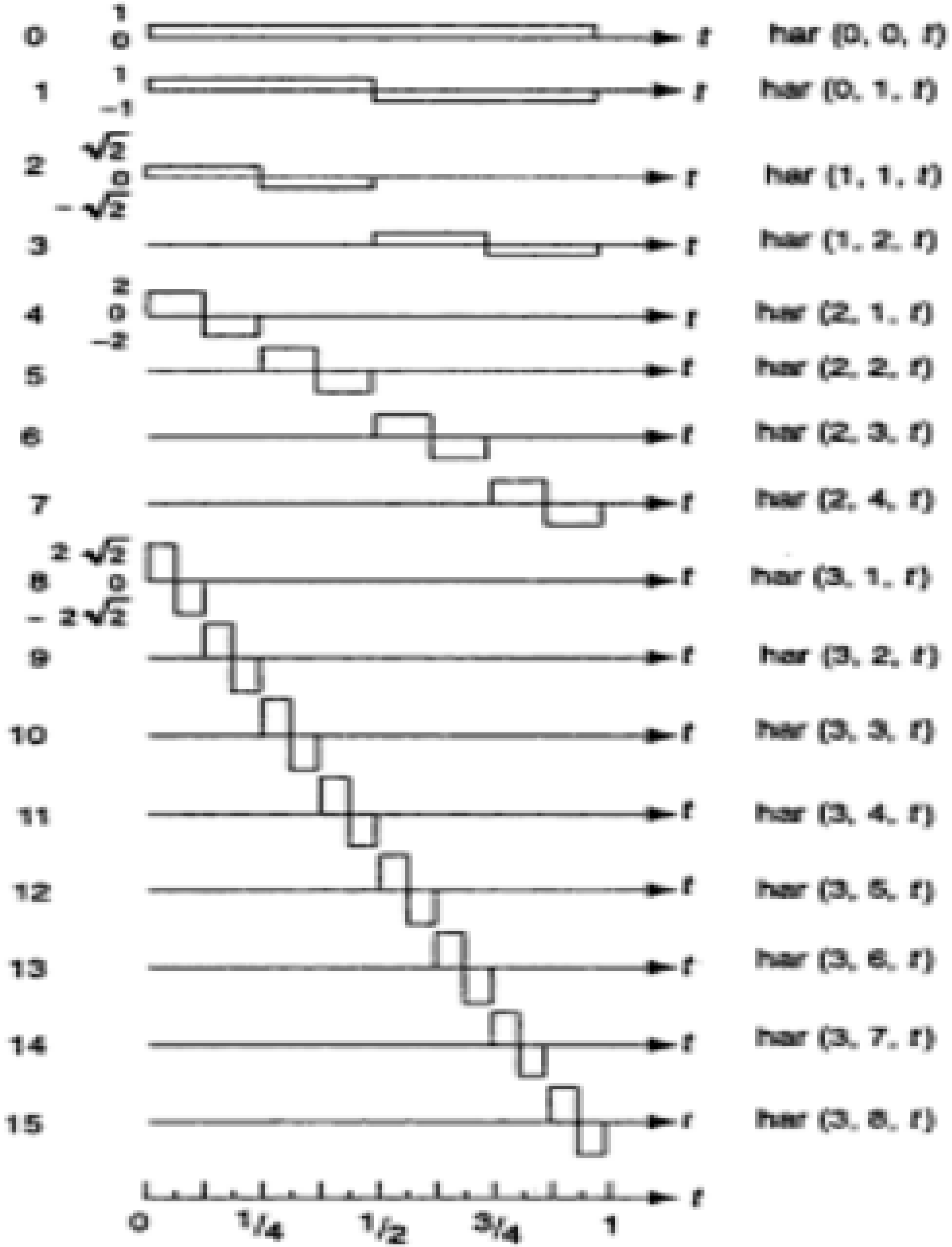
- Each row of the haar matrix is a discrete Haar function
- $[H_a(n)]$ is orthogonal i.e

$$[H_a(n)][H_a(n)]^T = NU_N$$

- The Haar transform and its inverse are

$$[C_h] = \frac{1}{N}[H_a(n)][X]$$

$$[X] = [H_a(n)]^T[C_h]$$


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
har (0, 0, t)
har (0, 1, t)
har (1, 1, t)
har (1, 2, t)
har (2, 1, t)
har (2, 2, t)
har (2, 3, t)
har (2, 4, t)
har (3, 1, t)
har (3, 2, t)
har (3, 3, t)
har (3, 4, t)
har (3, 5, t)
har (3, 6, t)
har (3, 7, t)
har (3, 8, t)
t
0
1/4
1/2
3/4
1

$$[H_a(4)] = \left[\begin{array}{cccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & -\sqrt{2} & -\sqrt{2} \\
2 & 2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & -2 & -2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & -2 & -2 \\
2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\sqrt{2} & -2\sqrt{2}
\end{array}\right]$$

Rationalized haar transform(RHT)

- *Rationalized version of Haar transform*
- *Deleting the irrational numbers (power of $\sqrt{2}$)*
- *Preserves all the properties of Haar transform*
- *The RHT is defined as*

$$[C_{rh}] = [R_h(n)][X]$$

- *Where*

$$[C_{rh}] = \left[C_{rh0}, C_{rh1}, C_{rh2}, \ldots, C_{rh(N-1)}\right]^T$$

$$n = \log_2^N \quad \text{or} \quad N = 2^n$$

- *And for N=16 :*

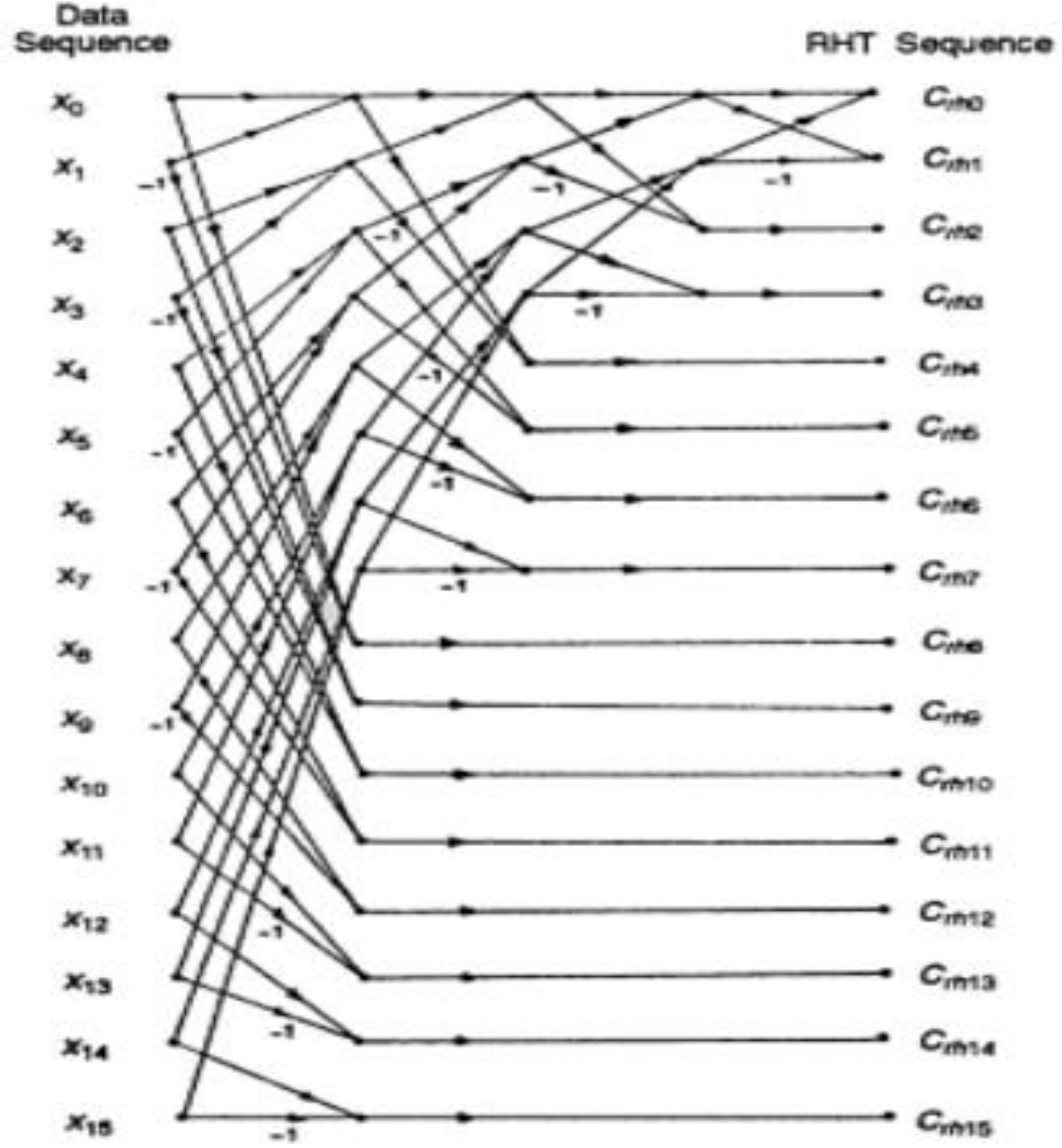
$$[R_h(4)] = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}$$

- *A fast algorithm for computation of RHT coefficients is based on matrix factorizing*

- *Based on this algorithm RHT can be implemented in 2(N-1) additions or subtractions*

$$[R_h(4)] = \left(diag\left[\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, U_{14}\right]\right)\left(diag\left[\begin{pmatrix} U_2 \otimes (1 & 1) \\ U_2 \otimes (1 & -1) \end{pmatrix}, U_{12}\right]\right)$$

$$\times\left(diag\left[\begin{pmatrix} U_4 \otimes (1 & 1) \\ U_4 \otimes (1 & -1) \end{pmatrix}, U_8\right]\right)\begin{pmatrix} U_8 \otimes (1 & 1) \\ U_8 \otimes (1 & -1) \end{pmatrix}$$



Data Sequence
RHT Sequence
x_0
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}
x_{11}
x_{12}
x_{13}
x_{14}
x_{15}
C_{rh0}
C_{rh1}
C_{rh2}
C_{rh3}
C_{rh4}
C_{rh5}
C_{rh6}
C_{rh7}
C_{rh8}
C_{rh9}
C_{rh10}
C_{rh11}
C_{rh12}
C_{rh13}
C_{rh14}
C_{rh15}
-1

Relatoinship between fourier and RHT coefficients

The fourier expantion:

$$x(t) = F_0 + \sqrt{2}F_1 \sin\frac{2\pi t}{T} + \sqrt{2}F_2 \cos\frac{2\pi t}{T}$$

$$\sqrt{2}F_3 \sin\frac{4\pi t}{T} + \sqrt{2}F_4 \cos\frac{4\pi t}{T} + \ldots$$

The above equations can be written in matrix form as

$$[F] = [S][X]$$

By substituting The fourier expantion in the the RHT coefficients equation:

$$[F] = [B][C_{rh}]$$

The fundamental fourier coefficients F1, F2 are as follows:

$$F_1 = 0.0555C_{rh1} - 0.011C_{rh2} + 0.011C_{rh3} - 0.0277C_{rh4}$$

$$0.0184C_{rh5} + 0.0277C_{rh6} - 0.0184C_{rh7}$$

$$F_2 = 0.011C_{rh1} + 0.0555C_{rh2} - 0.0555C_{rh3} + 0.0184C_{rh4}$$

$$0.0277C_{rh5} \_ 0.0184C_{rh6} - 0.0277C_{rh7}$$

$$[B] = \begin{bmatrix} 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0.0555 & -0.011 & 0.011 & -0.0277 & 0.0184 & 0.0277 & -0.0184 & \cdots \\ 0 & 0.011 & 0.0555 & -0.0555 & 0.0184 & 0.0277 & -0.0184 & -0.0277 & \cdots \\ \cdots & & & & \cdots & & & & \\ \cdots & & & & \cdots & & & & \\ \cdots & & & & \cdots & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \end{bmatrix}$$

Computation of apparent impedance

- ✓ Sampling both voltage and current signals over a full-cycle or half-cycle window (preferably half-cycle)
- ✓ Computation of RHT coefficients the fast algorithm
- ✓ Computation of the fundamental fourier coefficients(F1 and F2) by

$$F_1 = 0.0555C_{rh1} - 0.011C_{rh2} + 0.011C_{rh3} - 0.0277C_{rh4}$$

$$0.0184C_{rh5} + 0.0277C_{rh6} - 0.0184C_{rh7}$$

$$F_2 = 0.011C_{rh1} + 0.0555C_{rh2} - 0.0555C_{rh3} + 0.0184C_{rh4}$$

$$0.0277C_{rh5} \_ 0.0184C_{rh6} - 0.0277C_{rh7}$$

- ✓ then

$$F_{1(v)} = V_s, \qquad F_{2(v)} = V_c \qquad\Rightarrow\qquad V = V_s + jV_c$$

$$F_{1(i)} = I_s, \qquad F_{2(i)} = I_c \qquad\qquad I = I_s + jI_c$$

$$Z = \frac{V}{I} = \frac{|V|\angle\Phi_v}{|I|\angle\Phi_i} = \frac{\sqrt{V_s^2 + V_c^2}}{\sqrt{I_s^2 + I_c^2}}\angle(\Phi_v - \Phi_i)$$